Joint Power/Rate Optimization for CDMA-Based Wireless Sensor Networks*

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Abstract

In this paper, we address the problem of minimizing energy consumption on transmitting a certain number of information bits in a CDMA wireless sensor network (WSN). Both the transmission energy and signal-path circuit energy consumption are included in the objective function. The total energy consumption is minimized by jointly optimizing the transmission power and time for each active node in the network. For the numerical solution, we prove this formulation can be transformed to a convex geometric programming problem. For the analytical solution, we prove that the joint optimization on power and time can be decoupled into two sequential sub-problems: a parametric linear program in which the transmission time is a parameter, and a convex optimization problem to determine the optimal transmission time. Accordingly, closed-form solutions are found for both sub-problems and hence for the original formulation. Our results are verified through numerical examples and simulations.

1 Introduction

1.1 Motivations

Advances in mixed-signal design and microelectronic fabrication have made it possible to integrate analog and digital processing, sensing and wireless communication into a single integrated circuit. Such a device, when packaged with a battery and other electronics, forms a small, low cost sensor unit that can be easily deployed in large numbers to form a wireless sensor network (WSN). In the near future, WSNs will be utilized in a wide range of military and civilian applications, such as surveillance with object detection and tracking, environment and health monitoring, inventory tracking, failure detection, and many more [1]. The individual sensors, being powered by small batteries, have very limited energy capacity. Even in moderate size networks, replacement of batteries will not be feasible, either due to lack of access or very high cost. Consequently, strategies for achieving very high energy efficiency so as to maximize the lifetime of the network are essential.

Recently, it was established that the energy required to transmit a given amount of information increases exponentially as the transmission time decreases [3]. This simple transmission energy-delay tradeoff has been utilized in the design of energy-efficient packet scheduling protocols for single-user wireless links. In [4] and [5], the "lazy scheduling" approach was proposed. According to this approach, the energy used to transmit packets over a wireless link is minimized by judiciously varying packet transmission times according to the delay requirements. In [6] and [7], traffic smoothing is performed, resulting in an output packet traffic that is less bursty than the input traffic, and leading to significant power savings.

Although the tradeoff between transmission energy and transmission time has been extensively studied in the context of general wireless networks, such work is not directly applicable to WSNs due to specific features in node organization and transmission in a WSN. More specifically, because of the high density of nodes in a WSN, e.g., 20 nodes per meter^3 is not an unusual case [2], the average transmission distance between nodes is usually small. On the other hand, as more sophisticated computational and sensing functions are incorporated into a node, the circuit energy consumption will become increasingly significant. As a result, for such short-distance communications, the circuit energy consumption is no longer negligible relative to the transmission energy [13]. Therefore, a more complicated tradeoff emerges between energy and transmission time; although increasing the transmission time reduces the transmission energy, it also increases the circuit energy consumption. Another important feature that distinguishes a WSN from traditional wireless networks is the high correlation between nodes in a WSN. Because WSNs are often designed to cooperate on executing some

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joint task, less emphasis is put on per-node fairness. Accordingly, it is more reasonable to minimize the *total* energy consumption in the network instead of minimizing the energy consumption of individual nodes, i.e., a multi-user environment is more preferable for the optimization. Embracing the impact of circuit energy consumption and the new context of multiple access optimization, a new formulation is necessary to minimize the overall energy consumption in a WSN.

1.2 Related Works

A commonly studied approach for improving energy efficiency in WSNs is based on incorporating a "sleep" mode in the MAC (medium access control) operation (see, for example, [8, 9, 10, 11] and the references therein). This approach is motivated by the low duty cycle of sensors in typical deployment scenarios. Sensors will turn off their radio during idle times or when other nodes are transmitting. Different implementations have been devised for the sensor "sleep" operation. For example, in the SMACS-EAR protocol [8] sensors are scheduled to wake up at random times. In the S-MAC protocol [10], nodes are periodically put into sleep. The STEM protocol [9] emulates a paging channel to wake up sensor by having a separate ultra low-power radio. The CSMA-based MAC scheme in [11] shortens the carrier sensing (listening) duration by turning off the radio during the backoff period.

In the above schemes, the energy gain is achieved by suppressing energy consumption in the non-transmission phase (e.g., while a node is listening to the channel). Some recent studies have focused on controlling the transmission parameters to significantly improve energy efficiency. For example, in [12] the optimal packet size that minimizes the transmit energy consumption for a WSN was studied. The authors in [13]-[14] exploited the tradeoff between transmission and circuit energy consumption to provide an "optimal" cross-layer coding-modulation scheme for a single link.

More recently, the focus has shifted towards energy conservation and protocol design for multi-access transmission. In [15] the authors proposed an energy-efficient hybrid TDMA/FDMA MAC protocol for WSNs. They provided an analytical expression for the optimal number of channels that achieves the lowest power consumption. The works in [16] and [17] improve upon [13]-[14] by extending energy minimization to the multi-access case using a variable-length TDMA scheme. This scheme, however, has two practical limitations. First, as any TDMA-based scheme, it requires strict synchronization among various nodes. Second, its variable-length time-slot assignment approach does not scale well in a dense WSN with many nodes.

1.3 Contributions and Paper Organizations

In this work, we consider the use of CDMA as the channel access mechanism for sensors in a WSN. We study the optimal joint power/time control that minimizes energy consumption in a CDMA-based WSN. Both the transmit and the circuit energy consumption are accounted for in this optimization. In our setup, sensors are allowed to transmit data simultaneously to a remote sink using different spreading (signature) codes. Such a setup was first proposed in [19]. More recently, it was used in [18] to study the interference-connectivity tradeoff and in [20] for an ALOHA MAC protocol. In contrast to the TDMA scheme proposed in [15]-[17], the drawbacks of time synchronization and variable-time slot allocation are not present in a CDMA WSN.

The main contribution of this paper is twofold. First, although the objective function and the constraints in the underlying optimization problem are not convex, by exploiting the special structure of the formulation we successfully develop both numerical and closed-form analytical solutions to this problem. Numerically, this formulation is converted to a posynomial optimization problem that can be accurately solved by using geometric programming (GP). Second, analytically, we prove that the problem of jointly optimizing the transmission power and transmission time can be decoupled into two separate sequential sub-problems. The first is a parametric linear program for optimizing the transmission power with the transmission time being a parameter, and the second is a convex optimization problem for finding the optimal transmission time. We present closed-form solutions to both sub-problems, and consequently, to the original problem. Numerical examples and simulations are presented to validate our results. We also demonstrate the significant energy savings achieved by joint transmission power/time optimization.

The rest of this paper is organized as follows. We describe the system model in Section II. We formulate the problem and present the geometric programming-based numerical solution in Section III. Section IV presents an approximate closed-form analytical solution to the energy-minimization problem. Section V presents numerical examples and simulations, and Section VI concludes our work.

2 Model Description

2.1 System Model

We consider a DS-CDMA-based WSN [19][20] that consists of a set of densely distributed sensor nodes $\mathbf{S}$. The nodes transmit their data to a remote base station in a one-hop WSN or to a local cluster head in a clustered

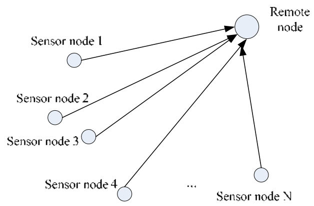

Figure 1: System model.

WSN [21]. Let o denote the destination node and let N be the number of active sensors at any given time instant, as illustrated in Figure 1. The information from the N sensors is transmitted simultaneously over a spread-spectrum bandwidth of W Hz. The single-sided power spectrum density of the additive white Gaussian noise (AWGN) is N_0 watt/Hz.

Per-cycle transmission power and transmission time control for all sensor nodes is performed by o. For sensor i ($i = 1, \ldots, N$), there are B_i bits in the queue waiting to be transmitted to o using transmit power P_{ti} and for a transmission duration T_i. Different transmission rates are supported by using variable spreading gains. Let the channel gain between nodes i and o be h_i and assume the channel is stationary for the duration T_i. The quality-of-service (QoS) requirement of sensor i is presented by the triple ($\gamma_i, T_i^{limit}, P_{\max}$), where γ_i is the minimum bit-energy-to-interference-ratio threshold for the received signal from sensor i, $T_i^{limit} \geq T_i$ is an upper limit on the transmission delay, and $P_{\max} \geq P_{ti}$ is the maximum transmit power (assumed the same for all nodes). As is common in DS-CDMA systems, we assume BPSK modulation. We must point out that, although we assume a common $P_{\max}$ for all nodes and BPSK modulation for the system, the analysis presented is not limited to these specific assumptions, and the corresponding results can be easily extended to accommodate heterogeneous power constraints and higher modulation schemes.

Remark: The above system model is suitable for a wide range of practical WSNs, including clock-driven, event-driven, and inquiry-driven systems. For a clock-driven WSN, the remote node o periodically (e.g., with period T) broadcasts beacons to activate simultaneous data transmissions from all nodes in $\mathbf{S}$. In this case, $N = |\mathbf{S}|$ and $T_i^{limit} = T$. For an event-driven WSN, a subset of $\mathbf{S}$ is activated simultaneously by the occurrence of an event. The activated nodes begin to transmit their sensed data roughly at the same time. Depending on the type of sensed data, e.g., voice, video, etc., there may be different deadlines for the transmissions from different sensors. Such deadlines are captured by $T_i^{limit}, i = 1, \ldots, N$. For an inquiry-driven WSN, node o broadcasts the inquiry request to the set $\mathbf{S}$, leading to a response from those sensors that have the desired answers. For a real-time inquiry, the desired information is usually needed by a certain time limit T^{limit}.

2.2 Energy Consumption Model

Consider the ith sensor node with B_i backlogged bits. The energy consumption at this node consists of transmission energy consumption and circuit energy consumption, i.e.,

$$E_i = (P_{ti} + P_{ci})T_i, \tag{1}$$

where P_{ci} is the power consumed by the circuit at sensor i. Following a similar model to the one in [13], P_{ci} can be written as

$$P_{ci} = \alpha_i + (\frac{1}{\eta} - 1)P_{ti}, \tag{2}$$

where α_i is a transmit-power-independent component that accounts for the power consumed by the digital-to-analog converter, the signal filters, and the modulator. $P_{PAi} \stackrel{\text{def}}{=} (\frac{1}{\eta} - 1)P_{ti}$ is the power consumed by the power amplifier (PA), whose value is related to the transmission power via the efficiency of the PA η, where $\eta = \frac{P_{ti}}{P_{ti}+P_{PAi}}$. Physically, η is determined by the drain efficiency of the RF power amplifier and the modulation scheme [13][22]. Substituting (2) into (1), the energy consumption of sensor i is given by

$$\begin{aligned} E_i &= \frac{1}{\eta}P_{ti}T_i + \alpha_i T_i \\ &= \frac{1}{\eta}(P_{ti} + \alpha_{ciri})T_i, \end{aligned} \tag{3}$$

where $\alpha_{ciri} = \eta\alpha_i$ is defined as the equivalent circuit power consumption. For N active sensor nodes, the total energy consumption is

$$E_{total} = \sum_{i=1}^{N} E_i = \frac{1}{\eta}\sum_{i=1}^{N}(P_{ti} + \alpha_{ciri})T_i. \tag{4}$$

3 Problem Formulation and Numerical Solution

The primary objective of our work is to find the optimal transmission power P_{ti}^o and transmission time T_i^o for each sensor node i such that the total energy consumption for transmitting $\sum_{i=1}^{N} B_i$ bits is minimized while the QoS requirement of each transmission is satisfied. Formally, this is expressed as

$$\begin{cases} \min_{\{\mathbf{P}_t,\mathbf{T}\}} \sum_{i=1}^{N}(P_{ti} + \alpha_{ciri})T_i \\ s.t. \\ \left(\frac{E_b}{I_0}\right)_i \geq \gamma_i, \quad i = 1, \ldots, N \\ 0 \leq T_i \leq T_i^{limit}, \quad i = 1, \ldots, N \\ 0 \leq P_{ti} \leq P_{\max}. \quad i = 1, \ldots, N \end{cases} \tag{5}$$

where $\mathbf{P}_t \stackrel{\text{def}}{=} (P_{t1}, \ldots, P_{tN})$ is the transmit power vector, $\mathbf{T} \stackrel{\text{def}}{=} (T_1, \ldots, T_N)$ is the transmission time vector, and $\left(\frac{E_b}{I_0}\right)_i$ is the received bit-energy-to-interference-density ratio at node o for sensor i. This $\left(\frac{E_b}{I_0}\right)_i$ is given by

$$\begin{aligned} \left(\frac{E_b}{I_0}\right)_i &= \frac{W}{R_i} \frac{h_i P_{ti}}{\delta \sum_{j=1, j\neq i}^{N} h_j P_{tj} + N_0 W} & (6) \\ &= \frac{W}{B_i} \frac{h_i P_{ti} T_i}{\delta \sum_{j=1, j\neq i}^{N} h_j P_{tj} + N_0 W} & (7) \end{aligned}$$

where $R_i = \frac{B_i}{T_i}$ is the transmission rate under the assumption of BPSK modulation and δ is the *orthogonality factor*, representing multiple access interference (MAI) from the imperfect-orthogonal spreading codes and the asynchronous chips across simultaneous transmitting nodes. Typical values for δ are $\frac{2}{3}$ and 1 for a chip of rectangular or sinoide shape, respectively. The second and third constraints in (5) come from the delay and transmit power upper bounds.

Because of the cross-product of $\mathbf{P}_t$ and $\mathbf{T}$ in the objective function and in the $\left(\frac{E_b}{I_0}\right)_i$ constraint, (5) is not a convex optimization problem. Hence, there is no guarantee that a locally optimal solution will indeed be globally optimal. We proceed to show that (5) can be put in a more *standard* form that reveals its special structure, for which an efficient numerical algorithm (geometric programming) is available. Moreover, as we show later, an approximate closed-form analytical solution is also possible due to the fact that the optimization problem can be solved sequentially, first with respect to power and then with respect to time.

Proposition 1: The problem formulation in (5) is a geometric programming, which can be transformed into a convex optimization problem of the so-called log-sum-exponential form so that the globally optimal solution can be efficiently derived by any numerical algorithm for convex optimization.

Proof: After some simple algebraic manipulations, (5) can be expressed as

$$\begin{cases} \min_{\{\mathbf{P}_t, \mathbf{T}\}} \sum_{i=1}^{N} (P_{ti} + \alpha_{ciri}) T_i \\ s.t. \\ \delta B_i \gamma_i (W h_i P_{ti} T_i)^{-1} \sum_{j=1, j\neq i}^{N} h_j P_{tj} \\ + B_i \gamma_i (W h_i P_{ti} T_i)^{-1} \leq 1, \quad i = 1, \ldots, N \\ \frac{T_i}{T_i^{limit}} \leq 1, \\ \frac{P_{ti}}{P_{\max}} \leq 1, \\ T_i \geq 0, \\ P_{ti} \geq 0. \end{cases} \quad (8)$$

The objective function and all of the left-hand sides of the constraints in (8) are sums of monomials in $(\mathbf{P}_t, \mathbf{T})$ with non-negative coefficients. These are known as posynomials[1], and (8) can be solved using geometric programming [23]. The above form is still not a convex optimization problem. However, with a transformation of variables, (8) can be converted into an equivalent convex optimization problem. Let $x_i = \ln P_{ti}$ and $y_i = \ln T_i$. Taking the logarithms of both the objective function and constraints, (8) is transformed into the following equivalent problem:

$$\begin{cases} \min_{\{\mathbf{x}, \mathbf{y}\}} \log \sum_{i=1}^{N} [\exp(x_i + y_i) + \exp(\ln \alpha_{ciri} + y_i)] \\ s.t. \\ \log \left[\sum_{j=1, j\neq i}^{N} \exp\left(x_j - x_i - y_i + \ln \delta B_i \gamma_i W^{-1} h_i^{-1} h_j\right)\right. \\ \left. + \exp\left(\ln(B_i \gamma_i W^{-1} h_i^{-1}) - x_i - y_i\right)\right] \leq 0 \\ \log \exp\left(y_i - \ln T_i^{limit}\right) \leq 0, \\ \log \exp\left(x_i - \ln P_{\max}\right) \leq 0, \quad i = 1, \ldots, N. \end{cases} \quad (9)$$

The log-sum-exponential function $f(\mathbf{z}) = \log\left(\sum_{i=1}^{n} e^{z_i}\right)$, where $\mathbf{z} = (z_1, \ldots, z_n) \in \mathbf{R}^n$, is a convex function [23]. This implies that the affine mapping $g(\mathbf{s}) = f(\mathbf{A}\mathbf{s} + \mathbf{B})$ preserves the convexity of $f(\mathbf{z})$. Hence, the objective function and all the constraints presented in (9) are convex, and so (9) is a convex optimization problem whose locally optimal solution $(\mathbf{x}^o, \mathbf{y}^o)$ is also globally optimal. Taking advantage of this useful property, efficient numerical algorithms for convex optimization problem, such as the primal-dual interior point method [23], can be used to solve for $(\mathbf{x}^o, \mathbf{y}^o)$. The globally optimal solution of (5) is simply given by $P_{ti}^o = \exp(x_i^o)$ and $T_i^o = \exp(y_i^o)$, for $i = 1, \ldots, N$. Thus, Proposition 1 follows. ■

Note that the transformation from the posynomial-form geometric program (8) to the convex-form problem (9) does not involve any computation; and the parameters for the two problems are the same. Therefore, the computational complexity is not increased by taking this transformation; it simply changes the form of the objective and constraint functions.

4 Closed-Form Analytical Results

The transformation of the optimization problem in (5) into (9) facilitates an accurate and very efficient numerical solution for finding the globally optimal transmission power and time for all active nodes in the system. In this section, we derive a closed-form analytical solution that may be viewed, in general, as a tight approximation of the exact solution. For all practical purposes, this analytical solution is indistinguishable from the numerical solution. The

[1]A posynomial in the variable $\mathbf{x} = (x_1, \ldots, x_n) \in \mathbf{R}^n$ is a linear combination of monomials with nonnegative coefficients. Formally, it is defined as $f(\mathbf{x}) = \sum_{k=1}^{K} c_k x_1^{a_{k1}} x_2^{a_{k2}} \ldots x_n^{a_{kn}}$, where $c_k \geq 0$ and $a_{kj} \in \mathbf{R}, j = 1, 2, \ldots, n$.

closed form of this solution makes it quite attractive for any real time transmit control operation.

The analytical solution is obtained by transforming the joint optimization problem in transmission power and time into two sequential sub-problems. This is achieved by first obtaining the optimal transmission power as an explicit function of the transmission time $\mathbf{T}$, for all feasible transmission times. Then, the optimal value of $\mathbf{T}$ is derived. Mathematically, this decoupling is described in the following section.

4.1 Sub-Problem 1: Parametric Solution for Optimal Transmission Power

Treating the transmission time vector $\mathbf{T}$ as a given system parameter with $T_i \leq T_i^{limit}$, problem (5) is equivalent to the following linear programming problem:

$$\begin{cases} \min_{\{P_{t1},\ldots,P_{tN}\}} \sum_{i=1}^N P_{ti} T_i \\ s.t. \\ \left(1+\frac{\delta B_i \gamma_i}{W T_i}\right) h_i P_{ti} - \frac{\delta B_i \gamma_i}{W T_i} \sum_{j=1}^N h_j P_{tj} \geq \frac{B_i \gamma_i N_0}{T_i}, \\ \qquad\qquad\qquad\qquad\qquad i = 1, \ldots, N \\ P_{ti} \leq P_{\max} \end{cases} \tag{10}$$

Regarding the optimal solution to (10), we have the following proposition.

Proposition 2: If the optimal solution to (10) exists, i.e., the feasible set depicted by the constraints in (10) is not empty, then this optimal solution is the solution to the following set of linear equations

$$\left(1+\frac{\delta B_i \gamma_i}{W T_i}\right) h_i P_{ti} - \frac{\delta B_i \gamma_i}{W T_i} \sum_{j=1}^N h_j P_{tj} = \frac{B_i \gamma_i N_0 W}{W T_i} \qquad i = 1, \ldots, N \tag{11}$$

Proof: Let $f_i(\mathbf{P}_t) \stackrel{\text{def}}{=} \left(1+\frac{\delta B_i \gamma_i}{W T_i}\right) h_i P_{ti} - \frac{\delta B_i \gamma_i}{W T_i} \sum_{j=1}^N h_j P_{tj}$, $i = 1, \ldots, N$. Its first-order partial derivations are

$$\frac{\partial f_i}{\partial P_{ti}} = h_i > 0 \tag{12}$$

and

$$\frac{\partial f_i}{\partial P_{tj}} = -\frac{B_i \gamma_i}{W T_i} h_j < 0, \quad \text{for } j \neq i. \tag{13}$$

The derivations indicate that $f_i(\mathbf{P}_t)$ is a strict mono-increasing function of P_{ti} and a strict mono-decreasing function of $P_{tj}, j \neq i$.

Let the optimal solution to (10) be $\mathbf{P}_t^o = (P_{t1}^o, \ldots, P_{tN}^o)$. Then it follows that $\sum_{i=1}^N P_{ti}^o T_i \leq \sum_{i=1}^N P_{ti} T_i$ for any feasible transmit power vector $\mathbf{P}_t = (P_{t1}, \ldots, P_{tN})$. Suppose for some node $k, 1 \leq k \leq N$, $f_k(\mathbf{P}_t^o) > \frac{B_k \gamma_k N_0}{T_k}$. Then, for this node, there must be some increment $\triangle P_{tk} > 0$ such that replacing P_{tk}^o by $P_{tk}^{o'} = P_{tk}^o - \triangle P_{tk}$ while keeping the transmit power of other nodes intact results in $f_k(\mathbf{P}_t^{o'}) \geq \frac{B_k \gamma_k N_0}{T_k}$ and $f_i(\mathbf{P}_t^{o'}) \geq \frac{B_i \gamma_i N_0}{T_i}$, for $i \neq k$, where $\mathbf{P}_t^{o'} = (P_{t1}^o, \ldots, P_{tk}^o - \triangle P_{tk}, \ldots, P_{tN}^o)$. Therefore, $\mathbf{P}_t^{o'}$ is also a feasible solution to problem (10). However, it is easy to show that $\sum_{i=1}^N P_{ti}^{o'} T_i$ is strictly smaller than $\sum_{i=1}^N P_{ti}^o T_i$ by $\triangle P_{tk} T_k$, leading to a conflict with the supposition that $\mathbf{P}_t^o$ is the optimal solution that minimizes the objective function $\sum_{i=1}^N P_{ti} T_i$. Therefore, there can not be any node k that does not meet the equality in the first constraint in (10). Then Proposition 2 follows. ■

After some mathematical manipulations of (11), we arrive at

$$h_i P_{ti} = \frac{\delta B_i \gamma_i}{W T_i + \delta B_i \gamma_i} \sum_{j=1}^N h_j P_{tj} + \frac{B_i \gamma_i}{W T_i + \delta B_i \gamma_i} N_0 W. \tag{14}$$

Define the *power index* of node i as:

$$g_i \stackrel{\text{def}}{=} \frac{\delta B_i \gamma_i}{W T_i + \delta B_i \gamma_i}, \tag{15}$$

Equation (14) can be rewritten as

$$h_i P_{ti} = g_i \sum_{j=1}^N h_j P_{tj} + \frac{1}{\delta} g_i N_0 W, \quad i = 1, \ldots, N. \tag{16}$$

Summing over i leads to

$$\sum_{i=1}^N h_i P_{ti} = g_\Sigma \sum_{j=1}^N h_j P_{tj} + \frac{1}{\delta} g_\Sigma N_0 W, \tag{17}$$

where $g_\Sigma \stackrel{\text{def}}{=} \sum_{i=1}^N g_i$. Therefore, the solution to (11), and also the optimal solution to problem (10) if it exists, is simply given by

$$P_{ti} = \frac{\delta^{-1} h_i^{-1} g_i}{1 - g_\Sigma} N_0 W. \tag{18}$$

Assuming $N_0 W = 1$, i.e., normalizing P_{ti} by the background AWGN, (18) is further simplified to

$$P_{ti} = \frac{\delta^{-1} h_i^{-1} g_i}{1 - g_\Sigma}, \quad i = 1, \ldots, N. \tag{19}$$

Given any feasible transmission time vector $\mathbf{T}$, (19) presents the optimal transmit power vector in terms of $\mathbf{T}$ if such optimal solution exists. Regarding the second constraint in (10), a necessary condition for the existence of the optimal solution is given by

$$P_{ti} = \frac{\delta^{-1} h_i^{-1} g_i}{1 - g_\Sigma} \leq P_{\max} \tag{20}$$

which leads to

$$g_i \leq \delta(1-g_\Sigma)h_i P_{\max}, \quad i=1,\ldots,N. \tag{21}$$

The inequality (21) depicts a polyhedron in $\mathbf{R}_+^N$ within which a feasible solution to (10) exists (thus, the optimal solution exists). Summing over i in (21), we have

$$g_\Sigma \leq \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma} < 1, \tag{22}$$

where $h_\Sigma \stackrel{\text{def}}{=} \sum_{i=1}^N h_i$. To provide a tractable closed-form solution, we relax (21) into

$$g_i \leq \delta h_i P_{\max}, \quad i=1,\ldots,N. \tag{23}$$

Note that this relaxation may result in transmission powers for some sensor nodes that exceed the upper bound $P_{\max}$ if the received signal quality constraints are to be satisfied for all nodes. However, for a typical CDMA-based WSN application, which is characterized by low data transmission rates, large spread spectrum bandwidth, and a relatively small SINR requirement, the g_i's are very small and $g_\Sigma \ll 1$. Consequently, the expansion of the feasible set through (23) will result in a tight approximation to the original polyhedron in (21), as will be demonstrated later in Section 6.

To summarize the results of this section, for any given feasible transmission time $\mathbf{T}$, the parametric optimal transmit power is given by (19). In order to guarantee the existence of this optimal power allocation, (23) and (22) must be satisfied, where g_i is defined in (15) and $g_\Sigma = \sum_{i=1}^N g_i$.

4.2 Sub-Problem 2: Optimization of Transmission Time

From (15), it is clear that for given B_i, γ_i, W, and δ, the power index g_i and the transmission time T_i are equivalent measures in the sense that there is a one-to-one mapping between g_i and T_i:

$$T_i = \frac{\delta B_i \gamma_i}{W g_i}(1-g_i). \tag{24}$$

In the following optimization, it is more mathematically convenient to work with g_i. Let $\mathbf{g} \stackrel{\text{def}}{=} (g_1,\ldots,g_N)$. The problem of determining the optimal value of $\mathbf{g}$ is formulated by substituting (24), (19), and the constraints (23) and (22) into the original optimization problem (5). This results in

$$\begin{cases} \min_{\{g_1,\ldots,g_N\}} \Big\{ h(g_1,\ldots,g_N) \stackrel{\text{def}}{=} \sum_{i=1}^N \Big(\frac{\delta^{-1}h_i^{-1}g_i}{1-g_\Sigma} + \alpha_{ciri} \Big) \\ \qquad\qquad \times \frac{\delta B_i\gamma_i}{W g_i}(1-g_i) \Big\} \\ s.t. \\ \frac{\delta B_i \gamma_i}{\delta B_i\gamma_i + W T_i^{limit}} \leq g_i \leq \delta h_i P_{\max}, \quad i=1,\ldots,N \\ \sum_{i=1}^N g_i \leq \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma} \end{cases} \tag{25}$$

where the lower bound on g_i in the first constraint comes from the delay bound requirement T_i. In most cases, (25) is a well-formulated problem, meaning that the upper bound requirement on g_i is larger than its lower bound, so the feasible solution set to (25) is not empty. However, in the case when both h_i and $P_{\max}$ are extremely small to the extent that the upper bound on g_i is smaller than its lower bound, the feasible set to (25) is null, and no solution exists to problem (5).

Rewriting the objective function $h(g_1,\ldots,g_N)$ in (25) by expanding the products results in

$$\begin{aligned} h(g_1,\ldots,g_N) &= \sum_{i=1}^N \frac{h_i^{-1}B_i\gamma_i(1-g_i)}{(1-g_\Sigma)W} \\ &+ \sum_{i=1}^N \frac{\alpha_{ciri}\delta B_i\gamma_i}{W g_i} - \sum_{i=1}^N \frac{\alpha_{ciri}\delta B_i\gamma_i}{W} \end{aligned} \tag{26}$$

As stated in the formulation of sub-problem 1, for a typical WSN application, $g_i \ll 1$. Therefore, (26) is tightly approximated by

$$\begin{aligned} h(g_1,\ldots,g_N) &\approx \frac{\sum_{i=1}^N h_i^{-1}B_i\gamma_i}{(1-g_\Sigma)W} + \sum_{i=1}^N \frac{\alpha_{ciri}\delta B_i\gamma_i}{W g_i} \\ &- \sum_{i=1}^N \frac{\alpha_i \delta B_i \gamma_i}{W} \\ &= \frac{K}{1-g_\Sigma} + \sum_{i=1}^N \frac{\alpha_{ciri}A_i}{g_i} - \sum_{i=1}^N \alpha_{ciri}A_i \end{aligned} \tag{27}$$

where $A_i \stackrel{\text{def}}{=} \frac{\delta B_i \gamma_i}{W}$ is a node-dependent constant and $K \stackrel{\text{def}}{=} \sum_{i=1}^N \delta^{-1} h_i^{-1} A_i$ is a system-dependent constant.

Proposition 3: The function $h(g_1,\ldots,g_N)$ in (27) is strictly convex.

Proof: The first-order partial derivative of $h(g_1,\ldots,g_N)$ with respect to g_i, $i=1,\ldots,N$, is given by

$$\frac{\partial h}{\partial g_i} = \frac{K}{(1-g_\Sigma)^2} - \frac{\alpha_{ciri}A_i}{g_i^2}. \tag{28}$$

The second-order partial deviation is given by

$$\frac{\partial^2 h}{\partial g_i^2} = \frac{2K}{(1-g_\Sigma)^3} + \frac{2\alpha_{ciri}A_i}{g_i^3}, \tag{29}$$

and for $i \neq j$

$$\frac{\partial^2 h}{\partial g_i \partial g_j} = \frac{2K}{(1-g_\Sigma)^3}. \tag{30}$$

Therefore, the *Heissian* of $h(g_1,\ldots,g_N)$ is given by[2]

$$\nabla^2 h(g_1,\ldots,g_N) = \frac{2K}{(1-g_\Sigma)^3}\mathbf{I} + \mathbf{D}, \tag{31}$$

[2]The element a_{ij} of the Heissian of a multi-variable function $f(x_1,\ldots,x_n)$ is defined as $a_{ij} = \frac{\partial^2 f}{\partial x_i \partial x_j}$, for $i,j=1,\ldots,n$.

where $\mathbf{I}$ is an $N \times N$ matrix with all elements equal to 1 and $\mathbf{D}$ is an $N \times N$ diagonal matrix whose ith diagonal element is $\frac{2\alpha_{ciri}}{g_i^3}$. For any non-zero vector $\mathbf{v} = (v_1, \ldots, v_N) \in \mathbf{R}^N$, it is easy to show that

$$\begin{aligned} \mathbf{v} \cdot \mathbf{I} \cdot \mathbf{v}^T &= \sum_{i=1}^{N} \sum_{j=1}^{N} v_i v_j \\ &= (v_1 + \ldots + v_N)^2 \geq 0, \end{aligned} \quad (32)$$

and

$$\mathbf{v} \cdot \mathbf{D} \cdot \mathbf{v}^T = \sum_{i=1}^{N} \frac{2\alpha_{ciri} A_i}{g_i^3} v_i^2 > 0. \quad (33)$$

Therefore, $\nabla^2 h(g_1, \ldots, g_N)$ is positive definite, and thus $h(g_1, \ldots, g_N)$ is a strictly convex function of $(g_1, \ldots, g_N)$. ■

Replacing $h(g_1, \ldots, g_N)$ in the objective function in (25) by its approximation in (27), we arrive at the following convex optimization problem

$$\begin{cases} \min_{\{g_1,\ldots,g_N\}} \frac{K}{1-g_\Sigma} + \sum_{i=1}^{N} \frac{\alpha_{ciri} A_i}{g_i} - \sum_{i=1}^{N} \alpha_{ciri} A_i \\ s.t. \\ \frac{\delta B_i \gamma_i}{\delta B_i \gamma_i + W T_i^{limit}} \leq g_i \leq \delta h_i P_{\max}, \quad i = 1, \ldots, N \\ \sum_{i=1}^{N} g_i \leq \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma}. \end{cases} \quad (34)$$

Since (27) is a tight approximation, we can also expect that the optimal solution to (34) will be a tight approximation to the optimal solution of (25).

The optimal solution $(g_1^o, \ldots, g_N^o)$ to the constrained problem (34) is related to the solution of the unconstrained minimization of $h(\mathbf{g})$. Being strictly convex, $h(g_1, \ldots, g_N)$ must have only one unconstrained minimum solution, which can be derived by solving the following equation set:

$$\frac{\partial h}{\partial g_i} = \frac{K}{(1-g_\Sigma)^2} - \frac{\alpha_{ciri} A_i}{g_i^2} = 0, \quad i = 1, \ldots, N. \quad (35)$$

Through some mathematical manipulations, it can be shown that the unconstrained optimum solution $(g_{u1}^o, \ldots, g_{uN}^o)$ to $h(g_1, \ldots, g_N)$ is given by

$$g_{ui}^o = \frac{\sqrt{\alpha_{ciri} A_i}}{\sqrt{K} + \sum_{i=1}^{N} \sqrt{\alpha_{ciri} A_i}}, \quad i = 1, \ldots, N. \quad (36)$$

Because of the convexity of $h(\mathbf{g})$, if any of the g_{ui}^o in (36) violates the upper or the lower bound on g_i in (34), then the corresponding constrained optimal solution g_i^o must itself be the upper or the lower bound, depending on which bound is being violated. Accordingly, the optimal solution to the constrained problem is given in the following proposition.

Proposition 4: Let $(g_1^o, \ldots, g_N^o)$ denote the optimal solution to (34). Let $g_i^{upp} \stackrel{\text{def}}{=} \delta h_i P_{\max}$ and $g_i^{low} \stackrel{\text{def}}{=} \frac{\delta B_i \gamma_i}{\delta B_i \gamma_i + W T_i^{limit}}$ be the upper and lower bounds on g_i, respectively. Let $\mathbf{V}$ denote the set of all active nodes, and let $\mathbf{U}$ denote the set of active nodes for which $g_i^o = g_i^{upp}$ or $g_i^o = g_i^{low}$. Define $t_1 \stackrel{\text{def}}{=} 1 - \sum_{j \in \mathbf{U}} g_j^o$ and $t_2 \stackrel{\text{def}}{=} \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma} - \sum_{j \in \mathbf{U}} g_j^o$. Then for $i = 1, \ldots, N$,

1. If $\sum_{i=1}^{N} g_i^o < \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma}$, then $g_i^o \in \left\{ g_i^{upp}, \frac{t_1 \sqrt{\alpha_i A_i}}{\sqrt{K} + \sum_{j \in \mathbf{V}-\mathbf{U}} \sqrt{\alpha_j A_j}}, g_i^{low} \right\}$.

2. If $\sum_{i=1}^{N} g_i^o = \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma}$, then $g_i^o \in \left\{ g_i^{upp}, \frac{t_2 \sqrt{\alpha_i A_i}}{\sum_{j \in \mathbf{V}-\mathbf{U}} \sqrt{\alpha_j A_j}}, g_i^{low} \right\}$.

Note: In either of these two cases, at least one g_i^o will equal the intermediate value.

Proof: The proof actually provides a recursive algorithm for solving for g_i^o.

Case 1: First, we consider the case when $\sum_{i=1}^{N} g_i^o < \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma}$. Let $\mathbf{U}$ be initially empty. Because of the strict convexity of $h(\mathbf{g})$, if for some i, the unconstrained optimal solution g_{ui}^o exceeds its upper bound, i.e., $g_{ui}^o > g_i^{upp}$, then the constrained optimal solution must be $g_i^o = g_i^{upp}$. Similarly, if $g_{ui}^o < g_i^{low}$, then $g_i^o = g_i^{low}$. Such nodes, whose unconstrained optimal solutions exceed their upper or lower bounds are added to the set $\mathbf{U}$. With the knowledge of g_i^o for $i \in \mathbf{U}$, the objective function in (34) is equivalent to the following function

$$\begin{aligned} h'(\mathbf{V}-\mathbf{U}) = \frac{K}{t_1 - g'_\Sigma} + \sum_{i \in \mathbf{V}-\mathbf{U}} \frac{\alpha_{ciri} A_i}{g_i} + \sum_{i \in \mathbf{U}} \frac{\alpha_{ciri} A_i}{g_i^o} \\ - \sum_{i=1}^{N} \alpha_{ciri} A_i, \end{aligned} \quad (37)$$

where $g'_\Sigma \stackrel{\text{def}}{=} \sum_{i \in \mathbf{V}-\mathbf{U}} g_i$. Because g_i^o is known for any $i \in \mathbf{U}$, replacing the objective function in (34) by (37) leads to an inherited problem that is of the same form as (34) except that the number of variables is reduced from $|\mathbf{V}|$ to $|\mathbf{V}-\mathbf{U}|$. With some mathematical manipulations, it can be shown that the unconstrained optimal solution to (37) is given by

$$g_{ui}^{o'} = \frac{t_1 \sqrt{\alpha_{ciri} A_i}}{\sqrt{K} + \sum_{j \in \mathbf{V}-\mathbf{U}} \sqrt{\alpha_{cirj} A_j}}, \quad i \in \mathbf{V}-\mathbf{U} \quad (38)$$

which is a recurrent version of (36) in terms of t_1 and $\mathbf{U}$. The above process is repeated and the values of t_1 and $\mathbf{U}$ are updated based on the newly computed values of g_i^o until all remaining unconstrained solutions g_{ui}^o, $i \in \mathbf{V}-\mathbf{U}$, of the inherited problem meet their respective upper and lower bounds. In the last iteration, the remaining g_i^o's, $i \in \mathbf{V}-\mathbf{U}$, are equal to their unconstrained counterparts given by (38).

Once all the g_i^o have been computed, it should be verified that $\sum_{i=1}^{N} g_i^o < \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma}$. If this is not the case, then the solution of g_i^o falls into the next case.

Case 2: Consider the case when $\sum_{i=1}^{N} g_i^o = \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma}$. In this case, the objective function in (34) degenerates into the following function

$$h_2(\mathbf{g}) \stackrel{\text{def}}{=} K(1+\delta P_{\max} h_\Sigma) + \sum_{i=1}^{N} \frac{\alpha_{ciri} A_i}{g_i} - \sum_{i=1}^{N} \alpha_{ciri} A_i. \tag{39}$$

Accordingly, (34) is equivalent to the following problem

$$\begin{cases} \min_{\{g_1,\ldots,g_N\}} \sum_{i=1}^{N} \frac{\alpha_{ciri} A_i}{g_i} \\ s.t. \\ \sum_{i=1}^{N} g_i = \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma}, \\ \frac{\delta B_i \gamma_i}{\delta B_i \gamma_i + W T_i^{limit}} \le g_i \le \delta h_i P_{\max}, \quad i = 1,\ldots,N. \end{cases} \tag{40}$$

In this case, it is easy to show that

$$\nabla^2 h_2(g_1,\ldots,g_N) = \text{diag}\left(\frac{2\alpha_{cir1} A_1}{g_2^3}, \ldots, \frac{2\alpha_{cirN} A_N}{g_N^3}\right), \tag{41}$$

which is a positive definite matrix. Therefore, $h_2(\mathbf{g})$ is a strictly convex function. Under the condition $\sum_{i=1}^{N} g_i^o = \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma}$, the optimal unbounded (i.e., ignoring the upper and lower bounds on g_i) solution to $h_2(\mathbf{g})$ is given by

$$g_{ui}^o = \frac{\frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma} \sqrt{\alpha_{ciri} A_i}}{\sum_{j=1}^{N} \sqrt{\alpha_{cirj} A_j}}. \tag{42}$$

Accounting for the upper- and lower-bound constraints of g_i and following a similar process to case 1, it can be found that g_i^o is equal to g_i^{upp}, g_i^{low}, or

$$g_i^o = \frac{t_2 \sqrt{\alpha_{ciri} A_i}}{\sum_{j \in \mathbf{V}-\mathbf{U}} \sqrt{\alpha_{cirj} A_j}}, \quad i \in \mathbf{V}-\mathbf{U}. \tag{43}$$

If in one of the computational cycles g_i^o is found to be equal to $\delta h_i P_{\max}$ or $\frac{\delta B_i \gamma_i}{\delta B_i \gamma_i + W T_{limit}^i}$ for all $i = 1,\ldots,N$, then there is no feasible solution to (34) because the constraint $\sum_{i=1}^{N} g_i^o = \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma}$ can not be satisfied. ∎

The above proof actually describes the "mechanics" for computing the optimal solution to (34). A pseudo-code representation of the computational algorithm is outlined in Table 1. The following example further illustrates the operation of this algorithm.

Example: Let $N = 5$, $K = 144$, $\alpha_{cir1} = \ldots = \alpha_{cir5} = 1$, $A_1 = A_2 = A_3 = 1$, $A_4 = 4$, $A_5 = 9$. The upper bounds are set to $g_i^{upp} = 0.1, 0.1, 0.055, 0.05, 0.1$ for $i = 1,\ldots,5$, respectively. Let $g_i^{low} = 0.01$ for all nodes, and let $\frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma} = 0.9$. To determine g_i^o for $i = 1,\ldots,5$, we first assume that $\sum_{i=1}^{5} g_i^o < 0.9$ and consider case 1 of Proposition 4 (once the g_i^o's have been computed, we

Initialization: For $i = 1,\ldots,N$, $A_i = \frac{\delta B_i \gamma_i}{W}$, $g_i^{upp} = \delta h_i P_{\max}$,
and $g_i^{low} = \frac{\delta B_i \gamma_i}{\delta B_i \gamma_i + W T_i^{limit}}$
$K = \sum_{i=1}^{N} \delta^{-1} h_i^{-1} A_i$, $t_1 = 1$, $t_2 = \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma}$
$\mathbf{V} = \{1,\ldots,N\}$, $\mathbf{U} = \emptyset$,
and *flag-continue* = TRUE
For all $i \in \mathbf{V}-\mathbf{U}$
  $f_i^{(1)}(t_1,\mathbf{U}) = \frac{t_1\sqrt{\alpha_i A_i}}{\sqrt{K}+\sum_{j\in\mathbf{V}-\mathbf{U}}\sqrt{\alpha_j A_j}}$,
  $f_i^{(2)}(t_2,\mathbf{U}) = \frac{t_2\sqrt{\alpha_i A_i}}{\sum_{j\in\mathbf{V}-\mathbf{U}}\sqrt{\alpha_j A_j}}$
End for
$m = 1$ // start with case 1

Iteration: While *flag-continue* = TRUE, do
  flag-continue = FALSE
  For all $i \in \mathbf{V}-\mathbf{U}$, set $g_{ui}^o = f_i^{(m)}(t_m,\mathbf{U})$
  For all $i \in \mathbf{V}-\mathbf{U}$, do
    If $g_{ui}^o > g_i^{upp}$,
      Set $g_i^o = g_i^{upp}$
      $\mathbf{U} = \mathbf{U} \cup \{i\}$
      flag-continue = TRUE
    Else if $g_{ui}^o < g_i^{low}$,
      Set $g_i^o = g_i^{low}$, $\mathbf{U} = \mathbf{U} \cup \{i\}$,
      and *flag-continue* = TRUE
    End if-else
  End for
  Update t_m:
    If $m = 1$, $t_1 = 1 - \sum_{i\in\mathbf{U}} g_i^o$
    Else, $t_2 = \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma} - \sum_{j\in\mathbf{U}} g_j^o$
  Update $f_i^{(m)}(t_m,\mathbf{U})$ as in the initialization step
End while

If $\mathbf{U} = \mathbf{V}$, exit // no feasible solution
Else for all $i \in \mathbf{V}-\mathbf{U}$, set $g_i^o = g_{ui}^o$
If ($m == 1$ && $\sum_{i=1}^{N} g_i^o < \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma}$) or
($m == 2$ && $\sum_{i=1}^{N} g_i^o = \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma}$)
  output $(g_1^o,\ldots,g_N^o)$ and exit
Else // case 2
  Set $\mathbf{U} = \emptyset$, *flag-continue* = TRUE, $m = 2$,
  and go to **Iteration**

Table 1: Pseudo-code for computing the optimal solution for transmit power and time.

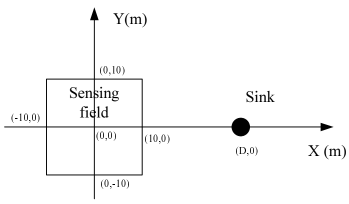

Figure 2: Sensing field used in the numerical examples.

can verify whether or not case 1 is the appropriate case). we initially set $\mathbf{U} = \emptyset$ and $t_1 = 1$.

In the first iteration, according to (38), we have $g^o_{u1} = g^o_{u2} = g^o_{u3} = 0.05$, $g^o_{u4} = 0.1$, and $g^o_{u5} = 0.15$. Comparing these values with their respective upper and lower bounds, we find that g^o_{u4} and g^o_{u5} violate their upper bounds. Therefore, we set $g^o_4 = g^{upp}_4 = 0.05$ and $g^o_5 = g^{upp}_5 = 0.1$ as their final values. Updating $\mathbf{U}$ and t_1, we have $\mathbf{U} = \{4, 5\}$ and $t_1 = 0.85$.

In the second iteration, we have $g^o_{u1} = g^o_{u2} = g^o_{u3} = 0.05667$. Comparing these values with their respective upper and lower bounds, we notice that g^o_{u3} violates its upper bound. Therefore, $g^o_3 = g^{upp}_3 = 0.055$. Updating $\mathbf{U}$ and t_1, we have $\mathbf{U} = \{3, 4, 5\}$ and $t_1 = 0.795$.

Finally, in the third iteration, we have $g^o_{u1} = g^o_{u2} = 0.0568$. Since both of these values are compliant with their upper and lower bounds, $g^o_1 = g^o_{u1} = 0.0568$ and $g^o_2 = g^o_{u2} = 0.0568$. After verifying that $\sum_{i=1}^5 g^o_i < 0.9$, the algorithm terminates.

Once the g^o_i's have been computed, the optimal transmit power and transmission time are obtained by combining (19), (24), and Proposition 4:

$$P^o_{ti} = \frac{\delta^{-1} h_i^{-1} g^o_i}{1 - g^o_\Sigma}, \tag{44}$$

$$T^o_i = \frac{\delta B_i \gamma_i}{W g^o_i}(1 - g^o_i), \quad i = 1, \ldots, N \tag{45}$$

where $g^o_\Sigma \stackrel{\text{def}}{=} \sum_{i=1}^N g^o_i$.

5 Numerical Investigations

In this section, we verify the accuracy of our analysis by comparing the analytical results obtained in Section IV with those of the numerical algorithm presented in Section III. The effect of relaxing the constraints and that of other approximations made in our analysis are also investigated.

5.1 System Settings

We consider a $20m \times 20m$ square sensing field, as shown in Figure 2, over which N homogeneous sensors are distributed uniformly. The sink node is located at $(D, 0)$. For each sensor node, the power amplifier energy efficiency is set to $\eta = 0.9$. The network is clock-driven and in every cycle of 1 second, all N sensors transmit their data simultaneously using DS-CDMA. A rectangular spreading chip is assumed, i.e. $\delta = \frac{2}{3}$. The threshold of the received SINR is 4 for all nodes. Each transmission must be completed within $T^{limit}_i = 1$ second. The spread spectrum bandwidth is $W = 1$ MHz and the single-sided power spectrum density of AWGN is $N_0 = 10^{-15}$ W/Hz. For sensor node i, the channel gain is given by

$$h_i = L(d_0) \left(\frac{d_i}{d_0} \right)^{-\mu} Y_i \left(X^2_{Ii} + X^2_{Qi} \right), \tag{46}$$

where $L(d_0) = \frac{G_t G_r \lambda^2}{16\pi^2 d_0^2}$ is the path loss of the close-in distance d_0, G_t and G_r are the antenna gains of the transmitter and the receiver, respectively, and λ is the wavelength of the carrier. We take $d_0 = 10$ meters and $G_t G_r = 1$. We also set the carrier frequency to 2.4 GHz. Let d_i be the distance between node i and the sink. The parameters Y_i, $i = 1, \ldots, N$, are i.i.d. lognormally distributed random variables with standard deviation 7dB. They account for the effect of shadowing. Moreover, X_{Ii} and X_{Qi} are the real and the imaginary parts of a Rayleigh fading channel gain, which follows a Gaussian distribution of mean zero and variance $\frac{1}{2}$. Finally, μ is the path loss exponent and is assumed to be 2 in our system, i.e., we consider a free-space loss model.

5.2 Numerical Results

In Figures 3-8, we examine the accuracy of our analysis by comparing the results obtained from the GP-based numerical algorithm and from the analytical algorithm proposed in Sections III and 4 under various network scales. For a given cycle, the channel gain of each node is generated according to (46). Both numerical and analytical algorithms are applied to calculate the optimal transmit power and transmission time for each node. The traffic generated by different nodes in each cycle is i.i.d. with a Poisson distribution of mean 100 bits. Although other, more realistic traffic models can be used in the simulations, this will have no impact on the qualitative (relative) performance of various optimization approaches. To illustrate the benefits of jointly optimizing transmit power and time, we also include in Figures 3, 5 and 7 the performance of a "fixed-transmission-time" strategy [3], whereby the transmission time for each sensor is set to the delay constraint ($1s$) and the transmit power is determined using (19). It can be observed that despite the approximate nature of our closed-form solution, this solution is almost indistinguishable from the GP-based numerical solution. This accuracy can be explained by noting that for a typical CDMA-based

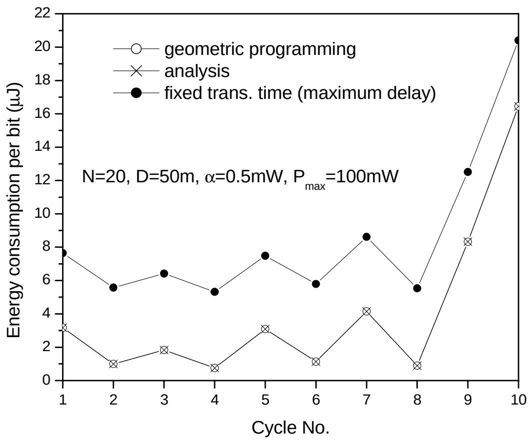

Figure 3: Trace of energy consumption per bit for ten successive cycles ($N = 20$).

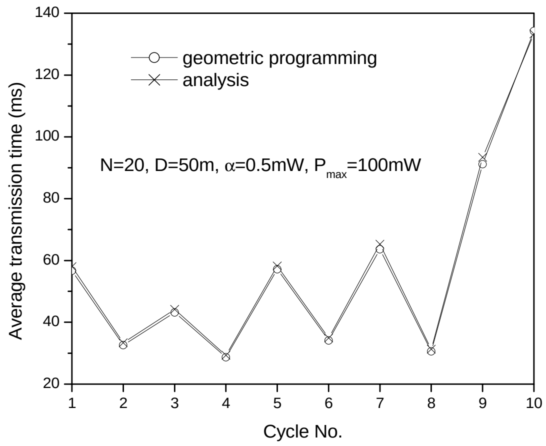

Figure 4: Trace of average sensor transmission time in ten successive cycles ($N = 20$).

WSN with a low data transmission rate, large spread spectrum bandwidth, and a small received SINR requirement, $g_i \ll 1$.

It should be noted, however, that the relaxation of the constraint on g_i from (21) into (23) may result in some nodes having optimal transmit powers greater than $P_{\max}$. Such nodes will obviously have to use $P_{\max}$ as their transmit power. Fortunately, this capping of power will only impact the the signal quality of such nodes (the SINR of other nodes will actually improve).

In Figures 7 and 8, we study the severity of violating the $P_{\max}$ constraint as a function of $P_{\max}$. We use two metrics for this purpose: *violation rate* and *violation degree*. The violation rate is defined as the average percentage of sensors in a cycle whose optimal transmit powers exceed $P_{\max}$. The violation degree is defined as the average power surplus over $P_{\max}$ required by those violating sensors. This value is normalized by $P_{\max}$. It is observed that for a wide range of N values (20 to 100), even under a tight power constraint of 10 mW, only a small percentage of sensors ($\approx 5\%$) violate the $P_{\max}$ constraint to a degree of 25%. Effectively, this says that in each transmission cycle, about 5% of the information bits are received at the sink below their SINR threshold with a normalized deficit of 0.25. Taking advantage of the rich data redundancy possessed by a WSN, the 5% data loss can be easily compensated for by other data transmitted from neighboring nodes. Using a more practical value for $P_{\max} = 100$ mW [13], the violation rate and degree are reduced to below 0.2% and 20%, respectively (over various values of N).

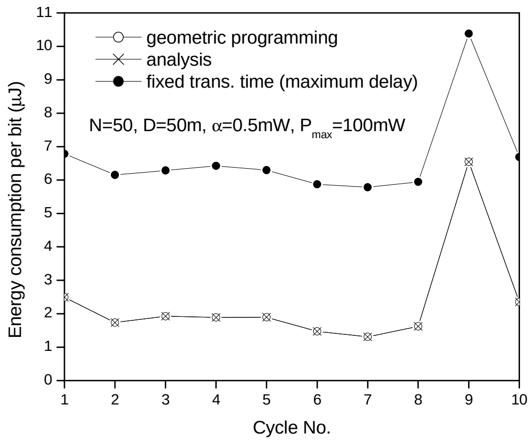

Figure 5: Trace of energy consumption per bit for ten successive cycles ($N = 50$).

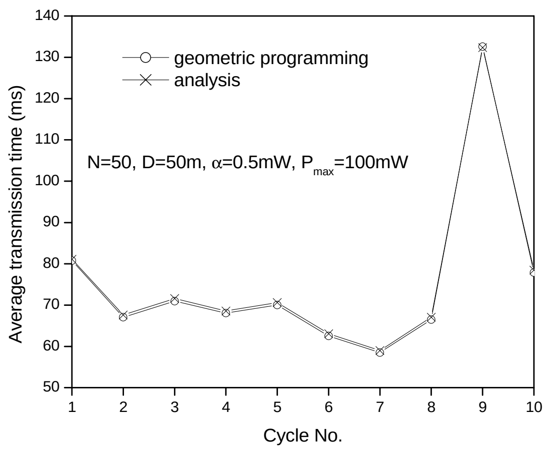

Figure 6: Trace of average sensor transmission time in ten successive cycles ($N = 50$).

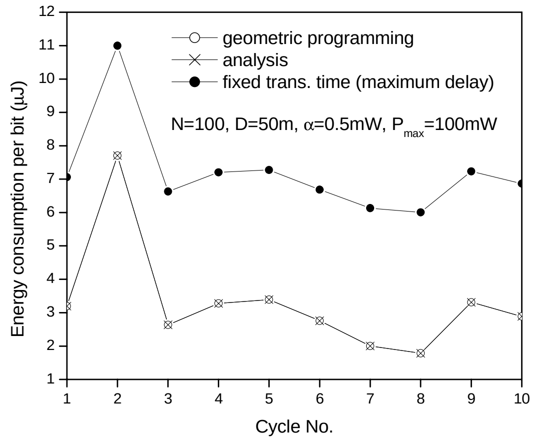

Figure 7: Trace of energy consumption per bit for ten successive cycles ($N = 100$).

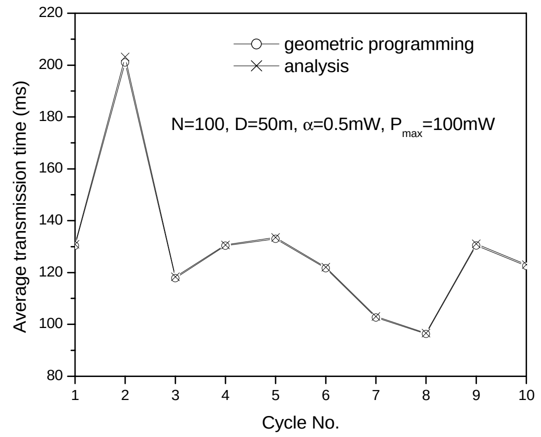

Figure 8: Trace of average sensor transmission time in ten successive cycles ($N = 100$).

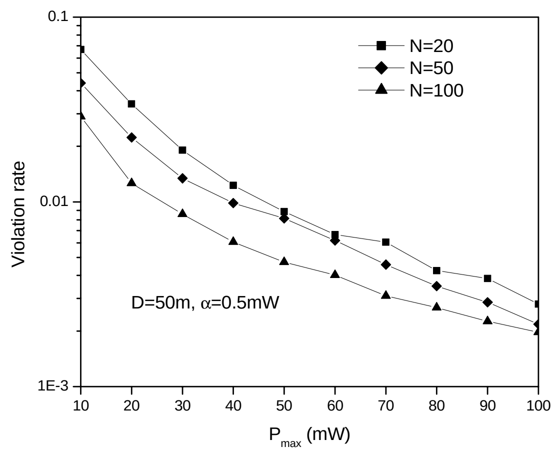

Figure 9: Violation rate of transmission power constraint vs. $P_{\max}$.

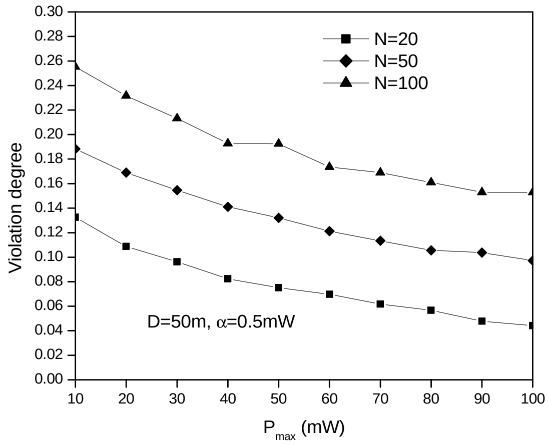

Figure 10: Violation degree of transmission power constraint vs. $P_{\max}$.

6 Summary

In this paper, we studied the problem of jointly optimizing the transmission powers and times of sensor nodes in a DS-CDMA WSN. The optimization was carried out for the purpose of minimizing the total energy consumption in the network. A comprehensive energy model was used, which accounts for both the transmit power consumption and the circuit energy consumption. The problem was formulated as a non-convex geometric program. In general, the non-convexity of the objective function and the constraints in such problems makes it quite challenging to obtain closed-form solutions. We first showed that the formulation can be transformed into a convex geometric program for which fast computational algorithms, such as the Interior Point Method, are applicable. Then, by exploiting the special structure of the underlying formulation, we derived a closed-form tight approximation for the optimal transmit powers and transmission times. Our closed-form solution is based on decoupling the optimization problem into two sequential sub-problems: a parametric linear program of the transmit power while transmission time serving as the parameter, and then an approximated convex program of the transmission time. The goodness of our solutions were verified through comparisons with simulation-based numerical results. These comparisons indicate that the closed-form expressions are extremely accurate, and can therefore be used as a basis for on-line real-time determining the optimal transmit power and times in a WSN.

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