Joint Optimization of Transmit Power-Time and Bit Energy Efficiency in CDMA Wireless Sensor Networks

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***Abstract*— In this paper, we address the problem of minimizing energy consumption in a CDMA wireless sensor network (WSN). A comprehensive energy consumption model is proposed, which accounts for both the transmit and circuit energies. Energy consumption is minimized by jointly optimizing the transmit power and transmission time for each active node in the network. The optimization problem is formulated as a non-convex optimization. Numerical as well as closed-form approximate solutions are provided. For the numerical solution, we show that the formulation can be transformed into a convex geometric programming (GP), for which fast algorithms, such as Interior Point Method, can be applied. For the closed-form solution, we prove that the joint power/time optimization can be decoupled into two sequential sub-problems: optimization of transmit power with transmission time serving as a parameter, and then optimization of the transmission time. We show that the first sub-problem is a linear programming while the second one can be well approximated as a convex programming problem. Taking advantage of these analytical results, we further derive the per-bit energy efficiency. Our results are verified through numerical examples and simulations.**

***Index Terms*— CDMA, sensor network, joint power and time optimization, geometric programming, convex optimization.**

I. Introduction

Wireless sensor networks (WSNs) are expected to be utilized in a wide range of military and civilian applications [1] in the near future. The sensors in these networks are typically powered by small batteries, which may be irreplaceable either due to lack of access or to prohibitive cost. Consequently, strategies for achieving high energy efficiency so as to maximize the lifetime of the network are essential.

It has been shown that the energy required to transmit a certain amount of information grows exponentially with the inverse of the transmission time [2]. This simple energy-delay tradeoff was applied in the design of energy-efficient packet scheduling protocols for single-user wireless links. For example, in [3] and [4], the "lazy scheduling" approach was proposed. According to this approach, the energy used to transmit packets over a wireless link is minimized by judiciously varying packet transmission times according to the delay requirements. In [5] and [6], traffic smoothing is performed, resulting in an output packet traffic that is less bursty than the input traffic, and leading to significant power savings.

Although the tradeoff between transmission energy and transmission time has been well studied for general wireless networks, such work is not directly applicable to WSNs due to two distinct features of these networks. First, because of the high density of nodes in a WSN, e.g., 20 nodes per meter^3 [1], the average transmission distance between nodes is usually small. For such short-range transmission, the circuit energy consumption is no longer negligible relative to the transmission energy [8]. Therefore, a more complicated tradeoff emerges between energy and transmission time; although increasing the transmission time reduces the transmission energy, it also increases the circuit energy consumption. Second, in a WSN, traffic streams from adjacent nodes exhibit a high degree of correlation. Because WSNs are often designed to cooperate on executing some joint task, less emphasis is usually put on per-node fairness. Accordingly, it is more reasonable to minimize the *total* energy consumption in the network instead of minimizing the energy consumption of individual nodes, i.e., a multi-user environment is more preferable for the optimization. By accounting for the impact of circuit energy consumption and the new context of multiple access optimization, a new formulation is necessary to minimize the overall energy consumption in a WSN.

Several previous studies incorporated circuit energy in the optimization of energy consumption for a single user. In [7] circuit energy consumption was included in the analysis of a cooperative and hierarchical WSN. In [8], the authors exploit the tradeoff between transmission and circuit energies to provide a cross-layer optimization of link-layer coding and physical-layer modulation for a single link. More recently, there has been some work on minimizing the total energy consumption in a multi-access environment. The authors in [9] improve upon the work in [8] by extending the point-to-point joint energy minimization to a multi-access scenario and presenting a variable-length Time Division Multiple Access (TDMA) scheme that minimizes the total energy consumption in the network. However, two major difficulties appear when implementing the ideas in [9], namely, the need for strict synchronization between different nodes and the scalability of the variable-length-time-slot allocation approach, especially in

This work was supported in part by the National Science Foundation through grants ANI-0095626, ANI-0313234, and ANI-0325979; and in part by the Center for Low Power Electronics (CLPE) at the University of Arizona. CLPE is supported by NSF (grant EEC-9523338), the State of Arizona, and a consortium of industrial partners. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

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a dense network.

In this work, we consider a CDMA-based WSN. Sensors are allowed to transmit data simultaneously to a remote sink using different spreading (signature) codes. The assumptions on time synchronization and variable-time slot allocation in [9] are not imposed. This setup was first proposed in [10] and was recently used in [11] and [12] to study the interference-connectivity tradeoff and MAC protocol design. In this paper, we study the joint control of transmit power and time to minimize the total network energy cost subject to constraints on the received signal quality, transmission delay, and transmission powers. Traditionally, joint power/rate control for CDMA cellular and ad hoc networks has been studied with the objective of minimizing the total transmit power (e.g., see [13], [14] and the references therein). In these studies, circuit power is negligible and a traffic of continuous bit stream is assumed. Our work differs from previous work in two fundamental features. First, to account for the low duty cycle of sensors, the traffic is represented by the number of bits to be transmitted in one duty cycle and the transmission time (duration) of these bits. Accordingly, we set energy, instead of power, as the optimization objective. Second, our energy consumption model accounts for both transmit and circuit energies.

The main contribution of this paper is twofold. First, although the objective function and the constraints in the underlying optimization problem are not convex, by exploiting the special structure of the formulation we successfully develop both numerical and closed-form analytical solutions to this problem. Numerically, this formulation is converted to a posynomial optimization problem that can be accurately solved by using geometric programming (GP). Analytically, we prove that the problem of jointly optimizing the transmission power and transmission time can be decoupled into two separate sequential sub-problems. The first is a parametric linear program for optimizing the transmission power with the transmission time being a parameter, and the second is a convex optimization problem for finding the optimal transmission time. We present approximate closed-form solutions to both sub-problems, and consequently, to the original problem. Taking advantage of the closed-form results, we further study the bit energy efficiency (BEE), defined as the minimum expected energy consumed to transmit a single information bit in the network while satisfying all constraints. For the special cases of WSNs with fully correlated or independent traffic from individual nodes, we obtain closed-form expressions or upper bounds on the BEE. Numerical examples and simulations are presented to validate our results.

The rest of this paper is organized as follows. We describe the system model in Section II. We formulate the problem and present the GP-based numerical solution in Section III. Section IV presents an approximate closed-form analytical solution to the energy-minimization problem. Based on this solution, we study the BEE performance for a CDMA-based WSN in Section V. Section VI presents numerical examples and simulations, and Section VII concludes our work.

II. Model Description

A. System Model

We consider a two-tired WSN [17], [18] that consists of two types of nodes. Type-I nodes are simple sensing nodes (SNs) that are responsible for sensing-related activities. Such nodes are small, low cost, and disposable, and can be densely deployed across the sensing area. Neighboring SNs are organized into clusters. Type-II nodes have more battery energy and stronger computational capability, and are referred to as cluster heads (CHs). We assume each CH is within the communication range of all SNs in its local cluster. Only limited communication functionalities are supported by a SN; it can transmit sensing data to or receive instructions from its CH, but cannot relay data from or instructions to a peer SN. Routing functions are supported by the CHs. A CH may collect data from the intra-cluster SNs, conduct signal processing on these raw data to create an application-specific view for the cluster, and then relay the fused data through intermediate CHs to the sink.

The above two-tiered structure is motivated by recent advances in distributed signal processing and source coding, which attempt to achieve a good balance among reliability, redundancy and scalability [19], [20]. Under this architecture, the goal of the lower-tier SNs and their CHs is primarily to gather data as effectively as possible; upper-tier CHs and the sink are designed to move information as efficiently as possible. The authors in [17] studied the energy efficiency of the upper tier through optimal selection of the sink location and inter-CH routing strategies. In this paper, we focus on the energy efficiency of the lower tier. We note that because of the large number of SNs, it is basically impractical to replace such nodes when their batteries are used up. In contrast, a CH may be replaceable, as relatively few CHs exist in the network. Improving the energy efficiency of the intra-cluster communication ultimately prolongs the lifetime of the whole network.

Without loss of generality, we focus on an arbitrary cluster that consists of a set $\mathbf{S}$ of densely distributed SNs. Let o denote the CH and let N be the number of active sensors in the cluster at any given time instant. The information from the N sensors is transmitted simultaneously over a spread-spectrum bandwidth of W Hz. The single-sided power spectrum density of the additive white Gaussian noise (AWGN) is N_0 watt/Hz. We assume that communications of different clusters can be distinguished by using perfectly orthogonal codes (e.g., Walsh codes). So only intra-cluster multiple access interference is considered.

Per-cycle transmission power and transmission time control for the sensor nodes is performed by o. For sensor i, $i = 1, \ldots, N$, there are B_i bits in the queue waiting to be transmitted to o using transmit power P_{ti} and for a transmission duration T_i. Different transmission rates are supported by using variable spreading gains. Let the channel gain between nodes i and o be h_i and assume the channel is stationary for the duration T_i. The constraints on sensor i's transmission are presented by the triple ($\gamma_i, T_i^{limit}, P_{\max}$), where γ_i is the minimum bit-energy-to-interference-ratio threshold for the received signal from sensor i, $T_i^{limit} \geq T_i$ is an upper limit on the transmission delay, and $P_{\max} \geq P_{ti}$ is the maximum transmit power (assumed the same for all nodes). As is common in DS-CDMA systems, we assume BPSK modulation.

B. Energy Consumption Model

Consider the ith SN with B_i backlogged bits. The energy consumption at this node consists of transmission energy consumption and circuit energy consumption, i.e.,

$$E_i = (P_{ti} + P_{ci})T_i, \tag{1}$$

where P_{ci} is the power consumed by the circuit at sensor i. Following a similar model to the one in [8], P_{ci} can be written as

$$P_{ci} = \alpha_i + (\frac{1}{\eta} - 1)P_{ti}, \tag{2}$$

where α_i is a transmit-power-independent component that accounts for the power consumed by the digital-to-analog converter, the signal filters, and the modulator. $P_{PAi} \stackrel{\text{def}}{=} (\frac{1}{\eta} - 1)P_{ti}$ is the power consumed by the power amplifier (PA), whose value is related to the transmission power via the efficiency of the PA η, where $\eta \frac{P_{ti}}{P_{ti}+P_{PAi}}$. Physically, η is determined by the drain efficiency of the RF power amplifier and the modulation scheme [8][22]. Substituting (2) into (1), the energy consumption of sensor i is given by

$$E_i = \frac{1}{\eta}P_{ti}T_i + \alpha_i T_i = \frac{1}{\eta}(P_{ti} + \alpha_{ciri})T_i, \tag{3}$$

where $\alpha_{ciri} = \eta\alpha_i$ is defined as the equivalent circuit power consumption. For N active sensor nodes, the total energy consumption is

$$E_{total} = \sum_{i=1}^{N} E_i = \frac{1}{\eta}\sum_{i=1}^{N}(P_{ti} + \alpha_{ciri})T_i. \tag{4}$$

III. Problem Formulation and Numerical Solution

Our primary objective is to find the optimal transmission power P_{ti}^o and optimal transmission time T_i^o for each sensor node i such that E_{total} is minimized while the various transmission constraints are satisfied. Formally, this is expressed as

$$\begin{cases} \min_{\{\mathbf{P}_t,\mathbf{T}\}} \sum_{i=1}^{N}(P_{ti} + \alpha_{ciri})T_i \\ s.t. \\ \left(\frac{E_b}{I_0}\right)_i \geq \gamma_i, \quad i = 1, \dots, N \\ 0 \leq T_i \leq T_i^{limit}, \quad i = 1, \dots, N \\ 0 \leq P_{ti} \leq P_{\max}. \quad i = 1, \dots, N \end{cases} \tag{5}$$

where $\mathbf{P}_t \stackrel{\text{def}}{=} (P_{t1}, \dots, P_{tN})$, $\mathbf{T} \stackrel{\text{def}}{=} (T_1, \dots, T_N)$, and $\left(\frac{E_b}{I_0}\right)_i$ is the received bit-energy-to-interference-density ratio at node o for sensor i. This $\left(\frac{E_b}{I_0}\right)_i$ is given by

$$\left(\frac{E_b}{I_0}\right)_i = \frac{W}{B_i}\frac{h_i P_{ti} T_i}{\delta\sum_{j=1,j\neq i}^{N} h_j P_{tj} + N_0 W} \tag{6}$$

where $R_i \stackrel{\text{def}}{=} \frac{B_i}{T_i}$ is the transmission rate under the assumption of BPSK modulation and δ is the *orthogonality factor*, representing multiple access interference (MAI) from the imperfect-orthogonal spreading codes and the asynchronous chips across simultaneously transmitting nodes. Typical values for δ are $\frac{2}{3}$ and 1 for a chip of rectangular or sinoide shape, respectively.

Because of the cross-product of $\mathbf{P}_t$ and $\mathbf{T}$ in the objective function and in the $\left(\frac{E_b}{I_0}\right)_i$ constraint, (5) is not a convex optimization problem. Hence, there is no guarantee that a locally optimal solution will indeed be globally optimal. We proceed to show that (5) can be put in a more *standard* form that reveals its special structure, for which an efficient numerical algorithm (geometric programming) is available. Moreover, as we show later, an approximate closed-form analytical solution is also possible due to the fact that the optimization problem can be solved sequentially, first with respect to power and then with respect to time.

Proposition 1: The formulation in (5) is a GP that can be transformed into a convex optimization problem of the so-called log-sum-exponential form.

Proof: After some simple algebraic manipulations, (5) can be expressed as

$$\begin{cases} \min_{\{\mathbf{P}_t,\mathbf{T}\}} \sum_{i=1}^{N}(P_{ti} + \alpha_{ciri})T_i \\ s.t. \\ \delta B_i\gamma_i\left(W h_i P_{ti} T_i\right)^{-1}\sum_{j=1,j\neq i}^{N} h_j P_{tj} \\ \qquad + B_i\gamma_i\left(W h_i P_{ti} T_i\right)^{-1} \leq 1, \quad i = 1, \dots, N \\ 0 \leq \frac{T_i}{T_i^{limit}} \leq 1, \\ 0 \leq \frac{P_{ti}}{P_{\max}} \leq 1. \end{cases} \tag{7}$$

The objective function and all of the left-hand sides of the constraints in (7) are sums of monomials in $(\mathbf{P}_t, \mathbf{T})$ with nonnegative coefficients. These are known as posynomials[1], and (7) can be solved using geometric programming [23]. The above form is still not convex. However, with a transformation of variables, (7) can be converted into an equivalent convex optimization problem. Let $x_i = \ln P_{ti}$ and $y_i = \ln T_i$. Taking the logarithms of both the objective function and constraints, (7) is transformed into the following equivalent problem:

$$\begin{cases} \min_{\{\mathbf{x},\mathbf{y}\}} \log\sum_{i=1}^{N}\left[\exp(x_i + y_i) + \exp(\ln\alpha_{ciri} + y_i)\right] \\ s.t. \\ \log\left[\sum_{j=1,j\neq i}^{N}\exp\left(x_j - x_i - y_i + \ln\delta B_i\gamma_i W^{-1}h_i^{-1}h_j\right)\right. \\ \left. + \exp\left(\ln(B_i\gamma_i W^{-1}h_i^{-1}) - x_i - y_i\right)\right] \leq 0 \\ \log\exp\left(y_i - \ln T_i^{limit}\right) \leq 0, \\ \log\exp\left(x_i - \ln P_{\max}\right) \leq 0, \quad i = 1, \dots, N. \end{cases} \tag{8}$$

The log-sum-exponential function $f(\mathbf{z}) = \log\left(\sum_{i=1}^{n} e^{z_i}\right)$, where $\mathbf{z} = (z_1, \dots, z_n) \in \mathbf{R}^n$, is a convex function [23]. This implies that the affine mapping $g(\mathbf{s}) = f(\mathbf{A}\mathbf{s} + \mathbf{B})$ preserves the convexity of $f(\mathbf{z})$. Hence, the objective function and all the constraints presented in (8) are convex, and so (8) is a convex optimization problem whose locally optimal solution $(\mathbf{x}^o, \mathbf{y}^o)$ is also globally optimal. Taking advantage of

[1]A posynomial in the variable $\mathbf{x} = (x_1, \dots, x_n) \in \mathbf{R}^n$ is a linear combination of monomials with nonnegative coefficients. Formally, it is defined as $f(\mathbf{x}) = \sum_{k=1}^{K} c_k x_1^{a_{k1}} x_2^{a_{k2}} \dots x_n^{a_{kn}}$, where $c_k \geq 0$ and $a_{kj} \in \mathbf{R}$, $j = 1, 2, \dots, n$.

this useful property, efficient numerical algorithms for convex optimization problems, such as the primal-dual interior point method [23], can be used to solve for $(\mathbf{x}^o, \mathbf{y}^o)$. The globally optimal solution of (5) is simply given by $P_{ti}^o = \exp(x_i^o)$ and $T_i^o = \exp(y_i^o)$, for $i = 1, \ldots, N$. ∎

IV. Closed-Form Analytical Results

In this section, we derive a closed-form approximate analytical solution to the optimization problem in (5). For all practical purposes, this analytical solution is indistinguishable from the exact numerical solution. The closed form of this solution makes it quite attractive for any real-time transmit control operation.

The analytical solution is obtained by decoupling the joint optimization problem in transmission power and time into two sequential sub-problems. This is achieved by first obtaining the optimal transmission power as an explicit function of the transmission time $\mathbf{T}$, for all feasible transmission times. Then, the optimal value of $\mathbf{T}$ is derived. A detailed mathematical description and justification of this decoupling approach has been given in [21]. Taking advantage of this property, the analysis of (5) proceeds as follows.

A. Sub-Problem 1: Parametric Solution for Optimal Transmission Power

Treating the transmission time vector $\mathbf{T}$ as a given system parameter with $T_i \leq T_i^{limit}$, problem (5) is equivalent to the following linear programming problem:

$$\begin{cases} \min_{\{P_{t1},\ldots,P_{tN}\}} \sum_{i=1}^{N} P_{ti}T_i \\ s.t. \\ \left(1+\frac{\delta B_i\gamma_i}{WT_i}\right) h_i P_{ti} - \frac{\delta B_i \gamma_i}{WT_i}\sum_{j=1}^{N} h_j P_{tj} \geq \frac{B_i\gamma_i N_0}{T_i}, \\ P_{ti} \leq P_{\max}, \quad i=1,\ldots,N. \end{cases} \tag{9}$$

Proposition 2: If the optimal solution to (9) exists, i.e., the feasible set depicted by the constraints in (9) is not empty, then this optimal solution is the solution to the following set of linear equations

$$\left(1+\frac{\delta B_i\gamma_i}{WT_i}\right) h_i P_{ti} - \frac{\delta B_i \gamma_i}{WT_i}\sum_{j=1}^{N} h_j P_{tj} = \frac{B_i\gamma_i N_0 W}{WT_i}, \quad i=1,\ldots,N. \tag{10}$$

Proof: Let $f_i(\mathbf{P}_t) \stackrel{\text{def}}{=} \left(1+\frac{\delta B_i\gamma_i}{WT_i}\right) h_i P_{ti} - \frac{\delta B_i\gamma_i}{WT_i}\sum_{j=1}^{N} h_j P_{tj}$, $i = 1, \ldots, N$. Its first-order partial derivatives are $\frac{\partial f_i}{\partial P_{ti}} = h_i > 0$ and $\frac{\partial f_i}{\partial P_{tj}} = -\frac{B_i\gamma_i}{WT_i}h_j < 0$ for $j \neq i$. The proof utilizes the property that $f_i(\mathbf{P}_t)$ is a strict mono-increasing function of P_{ti} and a strict mono-decreasing function of P_{tj}, $j \neq i$. For details, see [21]. ∎

After some mathematical manipulations of (10), we derive

$$P_{ti} = \frac{\delta^{-1}h_i^{-1}g_i}{1-g_\Sigma}, \quad i=1,\ldots,N \tag{11}$$

where g_i is the *power index* of node i

$$g_i \stackrel{\text{def}}{=} \frac{\delta B_i\gamma_i}{WT_i + \delta B_i\gamma_i} \tag{12}$$

and $g_\Sigma \stackrel{\text{def}}{=} \sum_{i=1}^{N} g_i$. For the sake of simplicity, P_{ti}'s in (11) and henceforth have been normalized with respect to the background AWGN, i.e., it is assumed that $N_0 W = 1$.

Given any feasible transmission time vector $\mathbf{T}$, (11) presents the optimal transmit power vector in terms of $\mathbf{T}$ if such an optimal solution exists. Regarding the second constraint in (9), a necessary condition for the existence of the optimal solution is given by

$$P_{ti} = \frac{\delta^{-1}h_i^{-1}g_i}{1-g_\Sigma} \leq P_{\max} \tag{13}$$

which leads to

$$g_i \leq \delta(1-g_\Sigma)h_i P_{\max}, \quad i=1,\ldots,N. \tag{14}$$

The inequality (14) depicts a polyhedron in $\mathbf{R}_+^N$ within which a feasible solution to (9) exists. Summing over i in (14), we have

$$g_\Sigma \leq \frac{\delta P_{\max}h_\Sigma}{1+\delta P_{\max}h_\Sigma} < 1, \tag{15}$$

where $h_\Sigma \stackrel{\text{def}}{=} \sum_{i=1}^{N} h_i$.

B. Sub-Problem 2: Optimization of Transmission Time

From (12), it is clear that for given B_i, γ_i, W, and δ, the power index g_i and the transmission time T_i are equivalent measures in the sense that there is a one-to-one mapping between g_i and T_i:

$$T_i = \frac{\delta B_i\gamma_i}{Wg_i}(1-g_i). \tag{16}$$

In the following optimization, it is more mathematically convenient to work with g_i. To provide a tractable closed-form solution, we relax (14) into

$$g_i \leq \delta h_i P_{\max}, \quad i=1,\ldots,N. \tag{17}$$

Note that this relaxation may result in some transmission powers exceeding the upper bound $P_{\max}$ if the received signal quality constraints are to be satisfied for all nodes. However, for a typical CDMA-based WSN application, which is characterized by low data transmission rates, large spread-spectrum bandwidth, and a low SINR requirement, it must be that $g_i \ll 1$ and g_Σ is considerably smaller than 1. As will be verified later in the numerical examples, the probability that the peak-power constranit is violated as a result of this relaxation is negiligible (e.g., less than 0.2% when $P_{\max} = 100$mW). Therefore, the expansion of the feasible set through (17) is a good approximation to the original polyhedron (14).

Let $\mathbf{g} \stackrel{\text{def}}{=} (g_1, \ldots, g_N)$. The problem of determining the optimal value of $\mathbf{g}$ is formulated by substituting (16), (11), and the constraints (17) and (15) into the original optimization problem (5). This results in

$$\begin{cases} \min_{\{\mathbf{g}\}} \left\{ h(\mathbf{g}) \stackrel{\text{def}}{=} \sum_{i=1}^{N}\left(\frac{\delta^{-1}h_i^{-1}g_i}{1-g_\Sigma} + \alpha_{ciri}\right)\frac{\delta B_i\gamma_i}{Wg_i}(1-g_i)\right\} \\ s.t. \\ \frac{\delta B_i\gamma_i}{\delta B_i\gamma_i + WT_i^{limit}} \leq g_i \leq \delta h_i P_{\max}, \quad i=1,\ldots,N \\ \sum_{i=1}^{N} g_i \leq \frac{\delta P_{\max}h_\Sigma}{1+\delta P_{\max}h_\Sigma} \end{cases} \tag{18}$$

where the lower bound on g_i in the first constraint comes from the delay bound requirement T_i.

Rewriting the objective function $h(g_1,\ldots,g_N)$ in (18) by expanding the products results in

$$h(\mathbf{g})=\sum_{i=1}^{N}\frac{h_i^{-1}B_i\gamma_i(1-g_i)}{(1-g_\Sigma)W}+\sum_{i=1}^{N}\frac{\alpha_{ciri}\delta B_i\gamma_i}{Wg_i}-\sum_{i=1}^{N}\frac{\alpha_{ciri}\delta B_i\gamma_i}{W}. \tag{19}$$

As stated earlier, for a typical WSN application, $g_i \ll 1$. Therefore, (19) is tightly approximated by

$$\begin{aligned} h(\mathbf{g}) &\approx \frac{\sum_{i=1}^{N} h_i^{-1}B_i\gamma_i}{(1-g_\Sigma)W}+\sum_{i=1}^{N}\frac{\alpha_{ciri}\delta B_i\gamma_i}{Wg_i}-\sum_{i=1}^{N}\frac{\alpha_i\delta B_i\gamma_i}{W}\\ &= \frac{K}{1-g_\Sigma}+\sum_{i=1}^{N}\frac{\alpha_{ciri}A_i}{g_i}-\sum_{i=1}^{N}\alpha_{ciri}A_i \end{aligned} \tag{20}$$

where $A_i \stackrel{\text{def}}{=} \frac{\delta B_i\gamma_i}{W}$ is a node-dependent constant and $K \stackrel{\text{def}}{=} \sum_{i=1}^{N}\delta^{-1}h_i^{-1}A_i$ is a system-dependent constant.

Proposition 3: The function $h(g_1,\ldots,g_N)$ in (20) is strictly convex.

Proof: The method is to prove that the *Heissian*[2] of $h(g_1,\ldots,g_N)$ is positive definite, thus $h(\mathbf{g})$ is a strictly convex function of $\mathbf{g}$. For details, see [21]. ■

Replacing $h(g_1,\ldots,g_N)$ in the objective function in (18) by its approximation in (20), we arrive at the following convex optimization problem

$$\begin{cases} \min_{\{g_1,\ldots,g_N\}} \frac{K}{1-g_\Sigma}+\sum_{i=1}^{N}\frac{\alpha_{ciri}A_i}{g_i}-\sum_{i=1}^{N}\alpha_{ciri}A_i\\ s.t.\\ \frac{\delta B_i\gamma_i}{\delta B_i\gamma_i+WT_i^{limit}} \le g_i \le \delta h_i P_{\max}, \quad i=1,\ldots,N\\ \sum_{i=1}^{N} g_i \le \frac{\delta P_{\max}h_\Sigma}{1+\delta P_{\max}h_\Sigma}. \end{cases} \tag{21}$$

Since (20) is a tight approximation, we can also expect that the optimal solution to (21) will be a good approximation to the optimal solution of (18).

The optimal solution $(g_1^o,\ldots,g_N^o)$ to the constrained problem (21) can be related to the solution of the unconstrained minimization of $h(\mathbf{g})$. Being strictly convex, $h(g_1,\ldots,g_N)$ must have only one unconstrained minimum solution, which can be derived by solving the following equation set:

$$\frac{\partial h}{\partial g_i}=\frac{K}{(1-g_\Sigma)^2}-\frac{\alpha_{ciri}A_i}{g_i^2}=0, \quad i=1,\ldots,N. \tag{22}$$

Through some mathematical manipulations, it can be shown that the unconstrained optimum solution $(g_{u1}^o,\ldots,g_{uN}^o)$ to $h(g_1,\ldots,g_N)$ is given by

$$g_{ui}^o=\frac{\sqrt{\alpha_{ciri}A_i}}{\sqrt{K}+\sum_{i=1}^{N}\sqrt{\alpha_{ciri}A_i}}, \quad i=1,\ldots,N. \tag{23}$$

Because of the convexity of $h(\mathbf{g})$, if any of the g_{ui}^o in (23) violates the upper or the lower bound on g_i in (21), then the corresponding constrained optimal solution g_i^o must itself be the upper or the lower bound, depending on which bound is being violated. Accordingly, the optimal solution to the constrained problem must have the following structure.

Proposition 4: The necessary condition of the constrained optimal solution: Let $(g_1^o,\ldots,g_N^o)$ denote the optimal solution to (21). Let $g_i^{upp} \stackrel{\text{def}}{=} \delta h_i P_{\max}$ and $g_i^{low} \stackrel{\text{def}}{=} \frac{\delta B_i\gamma_i}{\delta B_i\gamma_i+WT_i^{limit}}$ be the upper and lower bounds on g_i, respectively. Let $\mathbf{V}$ denote the set of all active nodes, and let $\mathbf{U}$ denote the set of active nodes for which $g_i^o=g_i^{upp}$ or $g_i^o=g_i^{low}$. Define $t_1 \stackrel{\text{def}}{=} 1-\sum_{j\in\mathbf{U}} g_j^o$ and $t_2 \stackrel{\text{def}}{=} \frac{\delta P_{\max}h_\Sigma}{1+\delta P_{\max}h_\Sigma}-\sum_{j\in\mathbf{U}} g_j^o$. Then for $i=1,\ldots,N$,

1. If $\sum_{i=1}^{N} g_i^o < \frac{\delta P_{\max}h_\Sigma}{1+\delta P_{\max}h_\Sigma}$, then $g_i^o \in \left\{g_i^{upp}, \frac{t_1\sqrt{\alpha_i A_i}}{\sqrt{K}+\sum_{j\in\mathbf{V}-\mathbf{U}}\sqrt{\alpha_j A_j}}, g_i^{low}\right\}$.

2. If $\sum_{i=1}^{N} g_i^o = \frac{\delta P_{\max}h_\Sigma}{1+\delta P_{\max}h_\Sigma}$, then $g_i^o \in \left\{g_i^{upp}, \frac{t_2\sqrt{\alpha_i A_i}}{\sum_{j\in\mathbf{V}-\mathbf{U}}\sqrt{\alpha_j A_j}}, g_i^{low}\right\}$.

Note: In the second case, at least one g_i^o will equal the intermediate value.

Proof: The proof actually provides a recursive algorithm for solving for g_i^o.

Case 1: First, we consider the case when $\sum_{i=1}^{N} g_i^o < \frac{\delta P_{\max}h_\Sigma}{1+\delta P_{\max}h_\Sigma}$. Let $\mathbf{U}$ be initially empty. Because of the strict convexity of $h(\mathbf{g})$, if for some i, the unconstrained optimal solution g_{ui}^o exceeds its upper bound, i.e., $g_{ui}^o > g_i^{upp}$, then the constrained optimal solution must be $g_i^o = g_i^{upp}$. Similarly, if $g_{ui}^o < g_i^{low}$, then $g_i^o = g_i^{low}$. Such nodes whose unconstrained optimal solutions exceed their upper or lower bounds are added to the set $\mathbf{U}$. With the knowledge of g_i^o for $i\in\mathbf{U}$, the objective function in (21) is equivalent to the following function

$$h'(\mathbf{V}-\mathbf{U})=\frac{K}{t_1-g'_\Sigma}+\sum_{i\in\mathbf{V}-\mathbf{U}}\frac{\alpha_{ciri}A_i}{g_i}+\sum_{i\in\mathbf{U}}\frac{\alpha_{ciri}A_i}{g_i^o}-\sum_{i=1}^{N}\alpha_{ciri}A_i \tag{24}$$

where $g'_\Sigma \stackrel{\text{def}}{=} \sum_{i\in\mathbf{V}-\mathbf{U}} g_i$. Because g_i^o is known for all $i\in\mathbf{U}$, replacing the objective function in (21) by (24) leads to an inherited problem that is of the same form as (21) except that the number of variables is reduced from $|\mathbf{V}|$ to $|\mathbf{V}-\mathbf{U}|$. With some mathematical manipulations, it can be shown that the unconstrained optimal solution to (24) is given by

$$g_{ui}^{o'}=\frac{t_1\sqrt{\alpha_{ciri}A_i}}{\sqrt{K}+\sum_{j\in\mathbf{V}-\mathbf{U}}\sqrt{\alpha_{cirj}A_j}}, \quad i\in\mathbf{V}-\mathbf{U} \tag{25}$$

which is a recurrent version of (23) in terms of t_1 and $\mathbf{U}$. The above process is repeated and the values of t_1 and $\mathbf{U}$ are updated based on the newly computed values of g_i^o until all unconstrained solutions g_{ui}^o, $i\in\mathbf{V}-\mathbf{U}$, of the inherited problem meet their respective upper and lower bounds. In the last iteration, the remaining g_i^o's, $i\in\mathbf{V}-\mathbf{U}$, are equal to their unconstrained counterparts given in (25).

Once all the g_i^o's have been computed, it should be verified that $\sum_{i=1}^{N} g_i^o < \frac{\delta P_{\max}h_\Sigma}{1+\delta P_{\max}h_\Sigma}$. If this is not the case, then the solution of g_i^o falls into the next case.

Case 2: Consider the case when $\sum_{i=1}^{N} g_i^o = \frac{\delta P_{\max}h_\Sigma}{1+\delta P_{\max}h_\Sigma}$. In this case, the objective function in (21) degenerates into the following function

$$h_2(\mathbf{g}) \stackrel{\text{def}}{=} K(1+\delta P_{\max}h_\Sigma)+\sum_{i=1}^{N}\frac{\alpha_{ciri}A_i}{g_i}-\sum_{i=1}^{N}\alpha_{ciri}A_i. \tag{26}$$

[2]The element a_{ij} of the Heissian of a multi-variable function $f(x_1,\ldots,x_n)$ is defined as $a_{ij}=\frac{\partial^2 f}{\partial x_i \partial x_j}$, for $i,j=1,\ldots,n$.

Accordingly, (21) is equivalent to the following problem

$$\begin{cases} \min_{\{g_1,\ldots,g_N\}} \sum_{i=1}^{N} \frac{\alpha_{ciri} A_i}{g_i} \\ s.t. \\ \sum_{i=1}^{N} g_i = \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma}, \\ \frac{\delta B_i \gamma_i}{\delta B_i \gamma_i + W T_i^{limit}} \leq g_i \leq \delta h_i P_{\max}, \quad i = 1, \ldots, N. \end{cases} \tag{27}$$

In this case, it is easy to show that

$$\nabla^2 h_2(g_1, \ldots, g_N) = \mathrm{diag}(\frac{2\alpha_{cir1} A_1}{g_2^3}, \ldots, \frac{2\alpha_{cirN} A_N}{g_N^3}) \tag{28}$$

which is a positive definite matrix. Therefore, $h_2(\mathbf{g})$ is a strictly convex function. Under the condition $\sum_{i=1}^{N} g_i^o = \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma}$, the unconstrained optimal solution to $h_2(\mathbf{g})$ is given by

$$g_{ui}^o = \frac{\frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma} \sqrt{\alpha_{ciri} A_i}}{\sum_{j=1}^{N} \sqrt{\alpha_{cirj} A_j}}. \tag{29}$$

Accounting for the upper- and lower-bound constraints on g_i and following a similar process to case 1, it can be found that g_i^o is equal to g_i^{upp}, g_i^{low}, or

$$g_i^o = \frac{t_2 \sqrt{\alpha_{ciri} A_i}}{\sum_{j \in \mathbf{V}-\mathbf{U}} \sqrt{\alpha_{cirj} A_j}}, \quad i \in \mathbf{V} - \mathbf{U}. \tag{30}$$

If in one of the computational cycles, g_i^o is found to be equal to $\delta h_i P_{\max}$ or $\frac{\delta B_i \gamma_i}{\delta B_i \gamma_i + W T_{limit}^i}$ for all $i = 1, \ldots, N$, then there is no feasible solution to (21) because the constraint $\sum_{i=1}^{N} g_i^o = \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma}$ can not be satisfied. ■

Proposition 4 indicates the structure that the constrained optimal solution will follow. It should be noted that in each iteration there are multiple possible constructions of $\mathbf{U}$, i.e., different combinations of the nodes that violate their bounds can be used to construct $\mathbf{U}$ in each iteration. To determine the optimal solution, all possible constructions of $\mathbf{U}$ in each iteration need to be considered. However, as explained later in Section V, for a practical CDMA WSN, the number of variables whose unconstrained optimal solutions exceed the bounds is usually small. In this case, we can approximate the optimal solution by only evaluating the particular $\mathbf{U}$ that consists of the whole set of violating nodes in each iteration. A pseudo-code description of the computational algorithm is outlined in Table I. The accuracy of this approximation process is examined later using numerical examples.

Once the g_i^o's have been computed, the optimal transmit power and transmission time are obtained by combining (11), (16), and Proposition 4:

$$P_{ti}^o = \frac{\delta^{-1} h_i^{-1} g_i^o}{1 - g_\Sigma^o}, \tag{31}$$

$$T_i^o = \frac{\delta B_i \gamma_i}{W g_i^o}(1 - g_i^o), \quad i = 1, \ldots, N \tag{32}$$

where $g_\Sigma^o \stackrel{\text{def}}{=} \sum_{i=1}^{N} g_i^o$.

Initialization:

For $i = 1, \ldots, N$, $A_i = \frac{\delta B_i \gamma_i}{W}$, $g_i^{upp} = \delta h_i P_{\max}$,
and $g_i^{low} = \frac{\delta B_i \gamma_i}{\delta B_i \gamma_i + W T_i^{limit}}$
$K = \sum_{i=1}^{N} \delta^{-1} h_i^{-1} A_i$, $t_1 = 1$, $t_2 = \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma}$
$\mathbf{V} = \{1, \ldots, N\}$, $\mathbf{U} = \emptyset$, and *flag-continue* = TRUE
For all $i \in \mathbf{V} - \mathbf{U}$
  $f_i^{(1)}(t_1, \mathbf{U}) = \frac{t_1 \sqrt{\alpha_i A_i}}{\sqrt{K} + \sum_{j \in \mathbf{V}-\mathbf{U}} \sqrt{\alpha_j A_j}}$,
  $f_i^{(2)}(t_2, \mathbf{U}) = \frac{t_2 \sqrt{\alpha_i A_i}}{\sum_{j \in \mathbf{V}-\mathbf{U}} \sqrt{\alpha_j A_j}}$
End for
$m = 1$ // start with case 1

Iteration:

While *flag-continue* = TRUE, do
  flag-continue = FALSE
  For all $i \in \mathbf{V} - \mathbf{U}$, set $g_{ui}^o = f_i^{(m)}(t_m, \mathbf{U})$
  For all $i \in \mathbf{V} - \mathbf{U}$, do
    If $g_{ui}^o > g_i^{upp}$,
      Set $g_i^o = g_i^{upp}$
      $\mathbf{U} = \mathbf{U} \cup \{i\}$
      flag-continue = TRUE
    Else if $g_{ui}^o < g_i^{low}$,
      Set $g_i^o = g_i^{low}$, $\mathbf{U} = \mathbf{U} \cup \{i\}$,
      and *flag-continue* = TRUE
    End if-else
  End for
  Update t_m:
    If $m = 1$, $t_1 = 1 - \sum_{i \in \mathbf{U}} g_i^o$
    Else, $t_2 = \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma} - \sum_{j \in \mathbf{U}} g_j^o$
  Update $f_i^{(m)}(t_m, \mathbf{U})$ as in the initialization step
End while
If $\mathbf{U} = \mathbf{V}$, exit // no feasible solution
Else for all $i \in \mathbf{V} - \mathbf{U}$, set $g_i^o = g_{ui}^o$
If ($m == 1$ && $\sum_{i=1}^{N} g_i^o < \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma}$) or
($m == 2$ && $\sum_{i=1}^{N} g_i^o = \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma}$)
  output $(g_1^o, \ldots, g_N^o)$ and exit
Else // case 2
  Set $\mathbf{U} = \emptyset$, *flag-continue* = TRUE, $m = 2$,
  and go to **Iteration**

TABLE I

PSEUDO-CODE FOR COMPUTING THE OPTIMAL SOLUTION FOR THE TRANSMIT POWERS AND TIMES.

V. BIT ENERGY EFFICIENCY

Based on the expression for the optimal transmit power and transmission time derived in Section IV, the minimum expected energy consumption for transmitting one information bit in a DS-CDMA based WSN, termed the *bit-energy efficiency* (BEE), can be studied analytically. To proceed with our analysis, we focus our attention on a *homogeneous* clock-driven WSN, i.e., we take $\alpha_{ciri} = \alpha_{cir}$ and $\gamma_i = \gamma$ for all i. We further assume that the considered WSN is well designed in the sense that it does not operate at the boundary of its capacity, i.e., the load of the traffic is reasonably less than the network capacity so that the vast majority of the optimal transmit power and time allocation are located within the polyhedron depicted by the constraints of (5).

From (23), the optimal power index of node i is given by

$$g_i^o = \frac{\sqrt{\alpha_{cir} A_i}}{\sqrt{K} + \sum_{i=1}^{N} \sqrt{\alpha_{cir} A_i}}, \quad i = 1, \ldots, N. \tag{33}$$

Substituting (33) into (11) and (16), we obtain simplified closed-form expressions for the optimal transmit power and time:

$$P_{ti}^o = \frac{\sqrt{\alpha B_i}}{h_i\sqrt{\delta\sum_{j=1}^N h_j^{-1}B_j}} \quad (34)$$

$$T_i^o = \frac{\delta B_i\gamma}{W g_i^o}(1-g_i^o) \simeq \frac{\delta B_i\gamma\left(\sqrt{\delta^{-1}\sum_{j=1}^N h_j^{-1}B_j}+\sum_{j=1}^N\sqrt{\alpha_{cir}B_j}\right)}{W\sqrt{\alpha_{cir}B_i}} \quad (35)$$

Substituting (34) and (35) into (4), the minimum energy required for the transmission of $\sum_i^N B_i$ bits in a given transmission cycle is given by

$$E_{total}^{\min} = \frac{\gamma}{W\eta}\left(\sum_{i=1}^N B_i h_i^{-1} + 2\sqrt{\alpha_{cir}\delta}\sum_{i=1}^N\sqrt{\sum_{j=1}^N h_j^{-1}B_iB_j} + \alpha_{cir}\delta\sum_{i=1}^N\sum_{j=1}^N\sqrt{B_iB_j}\right). \quad (36)$$

Suppose that B_i and h_i, $i=1,\ldots,N$, are arbitrarily defined random variables. Taking the expectation of (36) with respect to B_i and h_i gives $\mathrm{E}\{E_{total}^{\min}\}$; the minimum expected energy consumption in one transmission cycle. In general, $\mathrm{E}\{E_{total}^{\min}\}$ can not be expressed in a closed form. However, as stated in Proposition 5, a tight upper bound can be obtained.

Proposition 5:

$$\mathrm{E}\{E_{total}^{\min}\} \le \frac{\gamma}{W\eta}\left(\sum_{i=1}^N \mathrm{E}\{B_i\}\mathrm{E}\{h_i^{-1}\} + \alpha_{cir}\delta N\sum_{i=1}^N\mathrm{E}\{B_i\} + 2\sqrt{\alpha_{cir}\delta}\sum_{i=1}^N\sqrt{\sum_{j=1}^N\mathrm{E}\{h_j^{-1}\}\mathrm{E}\{B_iB_j\}}\right). \quad (37)$$

Proof: Because the geometric average of a sequence of nonnegative numbers can not be larger than their arithmetic average, we have

$$\mathrm{E}\left\{\alpha_{cir}\delta\sum_{i=1}^N\sum_{j=1}^N\sqrt{B_iB_j}\right\} = \alpha_{cir}\delta\sum_{i=1}^N\sum_{j=1}^N\mathrm{E}\{\sqrt{B_iB_j}\} \le \alpha_{cir}\delta\sum_{i=1}^N\sum_{j=1}^N\frac{\mathrm{E}\{B_i\}+\mathrm{E}\{B_j\}}{2} = \alpha_{cir}\delta N\sum_{i=1}^N\mathrm{E}\{B_i\}. \quad (38)$$

According to Jensen's inequality, $\mathrm{E}\{f(x)\}\le f(\mathrm{E}\{x\})$ for a concave function f. Thus,

$$\mathrm{E}\left\{\sqrt{\sum_{j=1}^N h_j^{-1}B_iB_j}\right\} \le \sqrt{\mathrm{E}\left\{\sum_{j=1}^N h_j^{-1}B_iB_j\right\}} = \sqrt{\sum_{j=1}^N\mathrm{E}\left\{h_j^{-1}\right\}\mathrm{E}\{B_iB_j\}} \quad (39)$$

where we assume that the channel gain h_i is independent of B_i. Substituting (38) and (39) into the expectation of (36), (37) follows. ∎

If (37) is convergent, an upper bound on the BEE is obtained by dividing (37) over the average number of bits transmitted in one transmission cycle, i.e.,

$$\mathrm{BEE} \le \frac{\gamma}{W\eta\sum_{i=1}^N\mathrm{E}\{B_i\}}\left(\sum_{i=1}^N\mathrm{E}\{B_i\}\mathrm{E}\{h_i^{-1}\} + \alpha_{cir}\delta N\sum_{i=1}^N\mathrm{E}\{B_i\} + 2\sqrt{\alpha_{cir}\delta}\sum_{i=1}^N\sqrt{\sum_{j=1}^N\mathrm{E}\{h_j^{-1}\}\mathrm{E}\{B_iB_j\}}\right) \quad (40)$$

Further simplification of this upper bound as well as closed-form expressions of the BEE can be obtained for some special cases. For example, if the B_i's are i.i.d. random variables and N is large, it can be shown that (40) can be further simplified to a traffic-distribution-independent asymptotic upper bound

$$\mathrm{BEE}^{iid} \le \frac{\gamma}{W\eta}\left(\frac{\mathrm{E}\{G\}}{N} + \alpha_{cir}\delta N + 2\sqrt{\alpha_{cir}\delta\mathrm{E}\{G\}}\right) \quad (41)$$

where $G \overset{\text{def}}{=} \sum_{i=1}^N h_i^{-1}$ is the sum of the inverse of channel gains. On the other hand, if $B_1 = B_2 = \ldots = B_N = B$ (fully correlated traffic), then the BEE is given by

$$\mathrm{BEE}^{FC} = \frac{\gamma}{W\eta}\left(\frac{\mathrm{E}\{G\}}{N} + \alpha_{cir}\delta N + 2\sqrt{\alpha_{cir}\delta}\mathrm{E}\left\{\sqrt{G}\right\}\right). \quad (42)$$

VI. Numerical Investigations

A. System Setup

We consider a $20m\times 20m$ sensing field that is centered at the origin. The field contains N uniformly distributed sensors. The sink is located at $(D,0)$, and a star topology is assumed. We set $\eta=0.9$, $\delta=2/3$, $N_0=10^{-15}$ W/Hz, $\gamma_i=4$dB, and $W=1$ MHz. A clock-driven WSN is assumed with $T_i^{limit}=1$ second. For sensor node i, the channel gain is given by

$$h_i = L(d_0)\left(\frac{d_i}{d_0}\right)^{-\mu} Y_i\left(X_{Ii}^2+X_{Qi}^2\right), \quad (43)$$

where $L(d_0)$ is the path loss of the close-in distance d_0, d_i is the distance between node i and the sink, and $\mu=2$ is the path loss exponent (i.e., we consider a free-space loss model). We take $d_0=10$ meters and set the carrier frequency to 2.4 GHz. The parameters Y_i, $i=1,\ldots,N$, are i.i.d. lognormally distributed random variables with standard deviation of 7 dB. They account for the effect of shadowing. The parameters X_{Ii} and X_{Qi} are the real and the imaginary parts of a Rayleigh fading channel gain, which follows a Gaussian distribution of mean zero and variance $\frac{1}{2}$.

B. Numerical Results

Figures 1 and 2 depict the BEE and average transmission time per node for 10 successive cycles. In each cycle, the channel gain of a node is generated according to (43). Both numerical (GP based) and analytical algorithms are applied to calculate the optimal transmit power and transmission time for each node. The traffic generated by different nodes in each cycle is i.i.d. with a Poisson distribution of mean 100 bits. To illustrate the benefits of jointly optimizing the transmission power and time, we also include in Figure 1 the performance

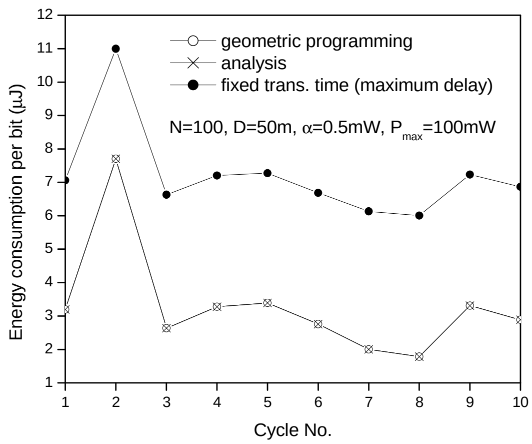

Fig. 1. Energy consumption per bit for ten successive cycles ($N = 100$).

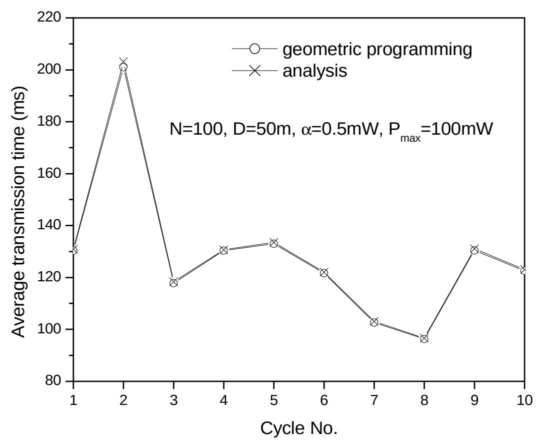

Fig. 2. Average transmission time in ten successive cycles ($N = 100$).

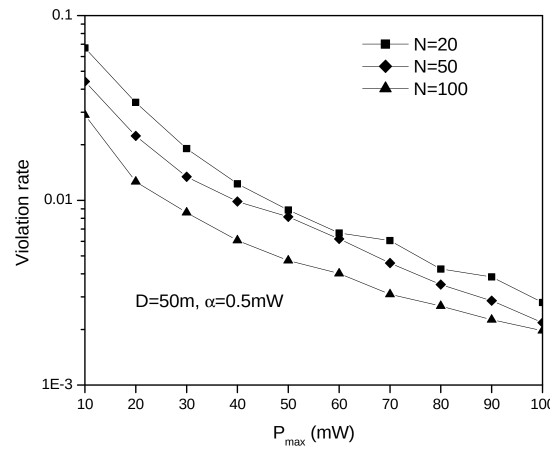

Fig. 3. Violation rate of transmission power constraint vs. $P_{\max}$.

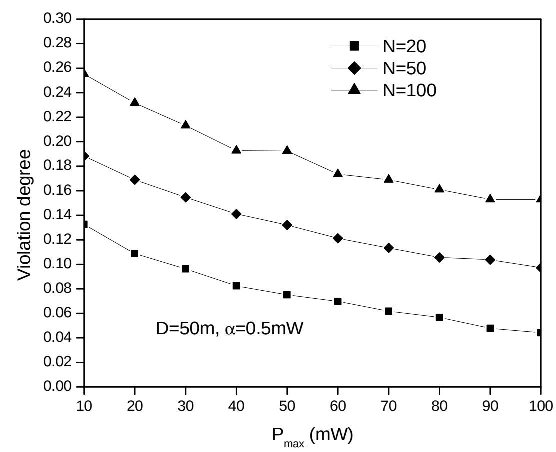

Fig. 4. Violation degree of transmission power constraint vs. $P_{\max}$.

of a "fixed-transmission-time" strategy [2], whereby the transmission time for each sensor is set to T_i^{limit} and the power is determined using (11). It can be observed that our approximate closed-form solution is almost indistinguishable from the GP-based numerical solution. From Figure 2, it can be seen that the average transmission duration of a node is 130 ms, which corresponds to $g_i \approx 2 \times 10^{-3} \ll 1$.

However, the relaxation of the constraint on g_i from (14) into (17) may result in some nodes having optimal transmit powers greater than $P_{\max}$. In practice, such nodes will have to use $P_{\max}$ as their transmit power. In Figures 3 and 4, we study the severity of violating the $P_{\max}$ constraint as a function of $P_{\max}$. We use two metrics for this purpose: *violation rate* and *violation degree*. The violation rate is defined as the percentage of sensors in a cycle whose optimal transmit powers exceed $P_{\max}$. The violation degree is defined as the average power surplus over $P_{\max}$ required by those violating sensors, normalized by $P_{\max}$. It is observed that for a wide range of N (20 to 100), even under a tight power constraint of 10 mW, only a small percentage of sensors ($\approx 5\%$) violate the $P_{\max}$ constraint to a degree of 25%. Effectively, this says that in each transmission cycle, about 5% of the information bits are received at the sink below their SINR threshold with a normalized deficit of 0.25. Taking advantage of the rich data redundancy of a WSN, the 5% data loss can be easily compensated for by other transmissions from neighboring nodes. Using a more practical value for $P_{\max} = 100$ mW [8], the violation rate and degree are reduced to below 0.2% and 20%, respectively (over various values of N).

In Figures 5 through 8, we study the BEE performance under various traffic scenarios. Figure 5 depicts the BEE versus N for the case of fully correlated traffic. The theoretical values (obtained from (42)) are compared with those from simulations where B_i is assumed to have a Poisson distribution with mean 100. In the simulations, the GP-based numerical algorithm is employed to determine the optimal transmit powers and times in each cycle. The figure shows that (42) accurately captures the BEE performance of a WSN. The case of i.i.d. traffic is considered in Figures 6-8, where the BEE is plotted as a function of the circuit power consumption (α), the remote node distance (D), and N, respectively. In these figures, we contrast the distribution-independent theoretical upper bound on the BEE (given in (41)) with three simulation-based BEE values that correspond to three different traffic distributions. The theoretical bound is found to be sufficiently tight. The simulation results also show that the BEE decreases with an increase in the variance of the traffic (compare the results for the cases $B_i \sim \text{uniform}(50, 150)$ and $B_i \sim \text{uniform}(20, 180)$).

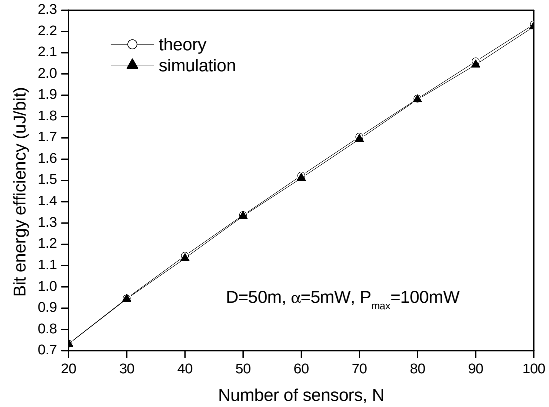

Fig. 5. BEE vs. N (fully correlated nodes).

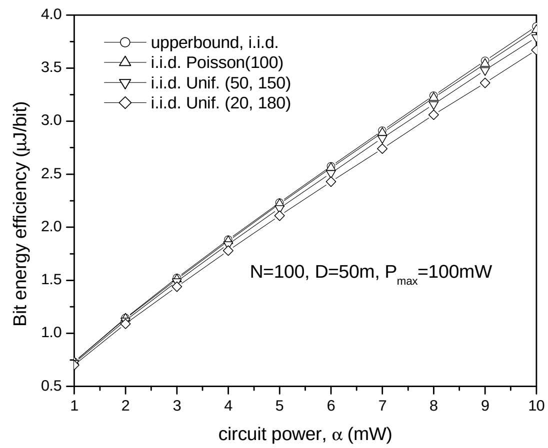

Fig. 6. BEE vs. circuit power (i.i.d. traffic).

This can be attributed, in part, to the nonlinearity of $E_{total}^{\min}$, given in (36). For example, consider the term B_1B_2 in (36). Under the constraint that $B_1 + B_2 = 2B$, where B is a constant, we have $B_1B_2 = -B_1^2 + 2BB_1$ where $0 \leq B_1 \leq 2B$. It is easy to see that B_1B_2 is a concave function for $0 \leq B_1 \leq 2B$, with its maximum value attained at $B_1 = B_2 = B$. This says that the function B_1B_2 is a mono-decrease function of the absolute difference between B_1 and B_2. Similarly, for a traffic distribution with a larger variation, the expected absolute difference between B_i and B_j in (36) will be larger, leading to a smaller product of B_iB_j, hence resulting in a smaller $E_{total}^{\min}$ and BEE.

VII. Conclusions

In this paper, we studied the problem of jointly optimizing the transmission powers and times of sensor nodes in a DS-CDMA WSN. The optimization was carried out for the purpose of minimizing the total energy consumption in the network. A comprehensive energy model was used, which accounts for both the transmit power consumption and the circuit energy consumption. The problem was formulated as a non-convex geometric program. In general, the non-convexity of the objective function and the constraints in such problems makes it quite challenging to obtain closed-form solutions. We first showed that the formulation can be transformed into a convex GP for which fast computational algorithms, such as the Interior Point Method, are applicable. By exploiting the special structure of the underlying formulation, we then derived a closed-form tight approximation for the optimal transmit powers and transmission times. Our closed-form solution is based on decoupling the optimization problem into two sequential sub-problems. First, we optimize the transmit powers, treating the transmission times as parameters. As a result of this step, the optimal powers are expressed as functions of the transmission times. In the second sub-problem, we optimize the transmission times. We showed that the first sub-problem is a linear program, while the second one is approximately a convex optimization problem. We further studied the bit energy efficiency for CDMA WSNs under various traffic scenarios. Closed-form expressions and bounds were obtained for the BEE. Comparisons with simulation results indicate that the closed-form expressions are extremely accurate, and can therefore be used as a basis for determining the optimal transmit power and times in a WSN. Our future work will focus on using such results in the design of protocols for dyanmic adjustment of the powers and times.

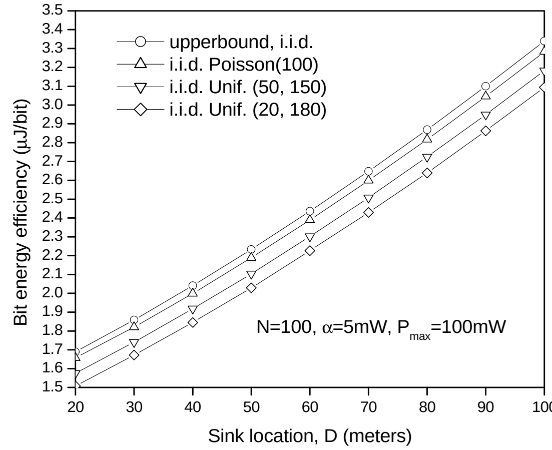

Fig. 7. BEE vs. sink location (i.i.d. traffic).

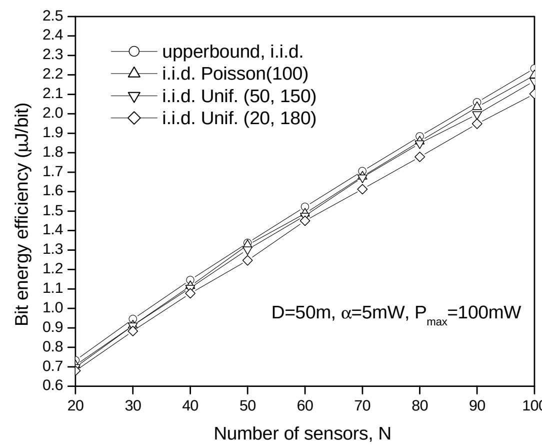

Fig. 8. BEE vs. N (i.i.d. traffic).

References

[1] I. F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "A survey on sensor networks," *IEEE Communications Magazine*, pp. 102-114,

Aug. 2002.

[2] R. Berry and R. Gallager, “Communication over fading channels with delay constraints,” *IEEE Trans. on Inform. Theory*, vol. 48, no. 5, pp. 1135-1149, May 2002.

[3] E. Uysal-Biyikoglu, B. Prabhakar, and A. E. Gamal, “Energy-efficient packet transmission over a wireless link,” *IEEE/ACM Transactions on Networking*, vol. 10, no. 4, pp. 487-499, Aug. 2002.

[4] A. E. Gamal, C. Nair, B. Prabhakar, E. Uysal-Biyikoglu, and S. Zahedi, “Energy-efficient scheduling of packet transmissions over wireless networks,” in *Proc. INFOCOM 2002*, vol. 3, pp. 1773-1783, June 2002.

[5] D. Rajan, A. Sabharwal, and B. Aazhang, “Delay bounded packet scheduling of bursty traffic over wireless channels,” *IEEE Transactions on Information Theory*, vol. 50, no. 1, pp. 125-144, Jan. 2004.

[6] M. A. Khojastepour and A. Sabharwal, “Delay-constrained scheduling: power efficiency, filter design, and bounds,” in *Proc. INFOCOM 2004*, March 2004.

[7] H. Dai, L. Xiao, and Q. Zhou, “On cooperative MIMO transmission strategies in wireless sensor networks with mobile agents,” invited paper, to appear in *2004 International Conference on Computing, Communications and Control Technologies*, Aug. 2004.

[8] S. Cui, A.J. Goldsmith, and A. Bahai, “Energy-constrained modulation optimization,” to appear in *IEEE Transactions on Wireless Communications*.

[9] S. Cui, A.J. Goldsmith, and A. Bahai, “Joint modulation and multiple access optimization under energy constraints,” in *Proc. IEEE Globecom’04*, vol. 1, pp. 151-155, Dec. 2004.

[10] C.H. Liu and H.H. Asada, “A source coding and modulation method for power saving and interference reduction in DS-CDMA sensor network systems,” in *Proc. American Control Conference 2002*, vol. 4, pp. 3003-3008, May 2002.

[11] P. Venkitasubramaniam, S. Adireddy, and L. Tong, “Sensor networks with mobile access: optimal random access and coding,” *IEEE Journal on Selected Areas in Communications*, vol. 22, no. 6, pp. 1058-1068, Aug. 2004.

[12] S. De, C. Qiao, D. A. Pados, M. Chatterjee, and S. J. Philip, “An integrated cross-layer study of wireless CDMA sensor networks,” *IEEE Journal on Selected Areas in Commun.*, vol. 22, no. 7, Sep. 2004, pp. 1271-1285.

[13] A. Sampath, P.S. Kumar, and J.M. Holtzman, “Power control and resource management for a multimedia CDMA wireless system,” in *Proc. IEEE Int. Symp. Personal, Indoor, Mobile Radio Communications (PIMRC’95)*, vol. 1, pp. 21-25, 1995.

[14] S. W. Kim and Y. H. Lee, “Combined rate and power adaptation in DS/CDMA communications over Nakagami fading channels,” *IEEE Trans. on Commun.*, vol. 48, no. 1, pp. 162-168, Jan. 2000.

[15] S. J. Lee and D. K. Sung, “Capacity evaluation for DS-CDMA systems with multi-class on/off traffic,” *IEEE Commun. Letters*, vol. 2, no. 6, pp. 153-155, June 1998.

[16] S. Bandyopadhyay and E. J. Coyle, “An energy efficient hierarchical clustering algorithm for wireless sensor networks,” in *Proc. IEEE INFOCOM 2003*, vol. 3, pp. 1713-1723, Mar. 2003.

[17] J. Pan, Y.T. Hou, L. Cai, Y. Shi, and S. X. Shen, “Topology control for wireless sensor networks,” in *Proc. ACM MobiCom 2003*, pp. 286-299, 2003.

[18] K. Romer and F. Mattern, “The design space of wireless sensor networks,” *IEEE Wireless Communications*, vol. 11, no. 6, pp. 54-61, Dec. 2004.

[19] J. Chou, D. Petrovis, and K. Ramchandran, “A distributed and adaptive signal processing approach to reducing energy consumption in sensor networks,” in *Proc. IEEE Infocom 2003*, pp. 1054-1062, April, 2003.

[20] S. S. Pradhan, J. Kusuma, and K. Ramchandran, “Distributed compression in a dense sensor network,” *IEEE Signal Processing Magazine*, vol. 19, no. 2, pp. 51-60, Mar. 2002.

[21] T. Shu, M. Krunz, and S. Vrudhula, “Joint optimization of transmit power-time and bit energy efficiency in CDMA wireless sensor networks,” University of Arizona, Department of ECE technical report, TR-UA-ECE-2005-3. http://www.ece.arizona.edu/∼krunz/TR/shu_joint_opt.pdf .

[22] T.H. Lee, *The Design of CMOS Radio-Frequency Integrated Circuits.* Cambridge Univ. Press, Cambridge, U.K., 1998.

[23] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge Univ. Press, Cambridge, U.K., 2004.

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