Spectrum Management and Power Allocation in MIMO Cognitive Networks

Diep N. Nguyen and Marwan Krunz
Department of Electrical and Computer Engineering, University of Arizona
E-mail:{dnnguyen, krunz}@ece.arizona.edu

***Abstract*— We consider the problem of maximizing the throughput of a multi-input multi-output (MIMO) cognitive radio (CR) network. CR users are assumed to share the available spectrum without disturbing primary radio (PR) transmissions. With spatial multiplexing performed over each frequency band, a multi-antenna CR node controls its antenna radiation patterns and allocates power for each data stream by appropriately adjusting its *precoding matrix*. Our objective is to design a set of precoding matrices (one for each band) at each CR node so that power and spectrum are optimally allocated for that node (in terms of throughput) and its interference is steered away from other CR and PR transmissions. In other words, the problems of power, spectrum and interference management are jointly investigated. We formulate a multi-carrier MIMO network throughput optimization problem subject to frequency-dependent power constraints. The problem is non-convex, with the number of variables growing quadratically with the number of antenna elements. Such a problem is difficult to solve, even in a centralized manner. To tackle it, we translate it into a noncooperative game and derive an optimal pricing policy for each node, which adapts to the node's neighboring conditions and drives the game to a Nash-Equilibrium (NE). The network throughput under this NE is at least equal to that of a locally optimal solution of the non-convex centralized problem. To find the set of precoding matrices at each node (the best response), a low-complexity distributed algorithm is developed by exploiting the strong duality of the per-user convex optimization problem. The number of variables in the distributed algorithm is independent of the number of antenna elements. A centralized (cooperative) algorithm is also developed, serving as a performance benchmark. Simulations show that the network throughput under the distributed algorithm converges rapidly to that of the centralized one. The fast convergence of the game facilitates MAC design, which we briefly discuss in the paper. The application of our results is not limited to CR systems, but extends to multi-carrier (e.g., OFDM) MIMO systems.**

***Index Terms*— Noncooperative game, pricing, cognitive radio, MIMO, MAC protocol, power allocation, frequency management, beamforming.**

I. Introduction

Cognitive radio (CR) and multi-input multi-output (MIMO) technologies have received great attention in recent years. While the former is viewed as a key enabling technology to improve spectrum utilization, the later has already proved itself as a powerful signal processing technique to improve spectral efficiency. Through sensing and/or probing, CRs can opportunistically communicate on temporarily available spectrum bands while avoiding interference with licensed-spectrum (or primary radio-PR) users. MIMO communications improve the channel capacity by sending independent data streams simultaneously over different antennas (a technique known as *spatial multiplexing*).

A crucial challenge in CR research is how to effectively allocate transmission powers and spectrum among CRs (see Figure 1(a)) so as to maximize network throughput while avoiding interfering with PR receptions. Even for a single frequency band and single-antenna wireless devices, the problem is difficult due to the non-convexity of the network throughput function. For single-antenna CRs, distributed algorithms were developed in [1] [2] using game theory.

The incorporation of MIMO techniques into CR systems introduces two new control dimensions (besides power control and frequency management): power allocation over antennas (space dimension) and interference management. The latter comes from MIMO's *degrees of freedom* [3], which allow a MIMO node to suppress interference from others (by using some of its antennas) and configure its antenna radiation patterns to keep interference away from unintended receivers. MIMO's power allocation and interference management can be jointly controlled via *precoding matrices*, a spatial multiplexing technique [3]. Using this technique, the vector of information symbols are pre-multiplied with a matrix before being placed on a transmit antenna array. By tuning the amplitude and the phase of each complex entry in the precoding matrix, one adjusts not only the allocated powers but also the radiation directions, which together shape the antenna radiation patterns. Previous MIMO-networking works [4] [5] [6] considered power allocation or *stream control* (Figure 1(b)) but did not take into account interference management via controlling antenna beams. An optimal set of precoding matrices for each node would be one that allocates power over *both* space and frequency dimensions (1(c)) *and* yields radiation patterns that induce minimum interference (1(d)), so as to maximize the network throughput. This problem is the focus of our work.

If one ignores the need to protect PR receptions, a MIMO-based CR system resembles a multi-carrier (e.g., OFDM) MIMO (MC-MIMO) system. In MC-MIMO, joint power and spectrum optimization is a non-convex problem, which is attributed to co-channel multi-user interference. Globally optimal solvers for non-convex problems, e.g., branch and bound, often have exponentially growing complexity in the number of variables. Unfortunately, the number of variables in a MC-MIMO network can be very large. For instance, when using the precoding technique with 4 antennas per node and 10 subcarriers in a network of 10 links, this number is $4 \times 4 \times 10 \times 10 = 1600$ complex variables (or 3200 real variables). The latest advances in power and spectrum management for MC-MIMO can be found in [7] using dual stochastic optimization. Similar to the aforementioned works, the authors in [7] [8] only considered the power allocation (the amplitude) but did not optimize the antenna radiation directions (the phase).

There have been recent works at the physical layer that attempt to protect PR communications in a MIMO CR network (CRN) while maximizing the CRN's throughput (e.g., [9] [10] [11]). These works assume full or partial availability of channel state information (CSI) from each CR to each PR. This requires feedback or coordination between CRs and PRs. However, current licensed radio devices are not ready for such a feedback mechanism, as CR communications are expected to be transparent to PRs. In our setup, PR communications are protected by imposing frequency-dependent constraint on the transmission power of CRs. This assumption is widely used in single-antenna CRNs e.g., [2] [12]. It should be noted that our

This research was supported in part by NSF (under grants CNS-1016943, CNS-0721935, CNS-0904681, IIP-0832238), Raytheon, and the Connection One center. Any opinions, findings, conclusions, or recommendations expressed in this paper are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

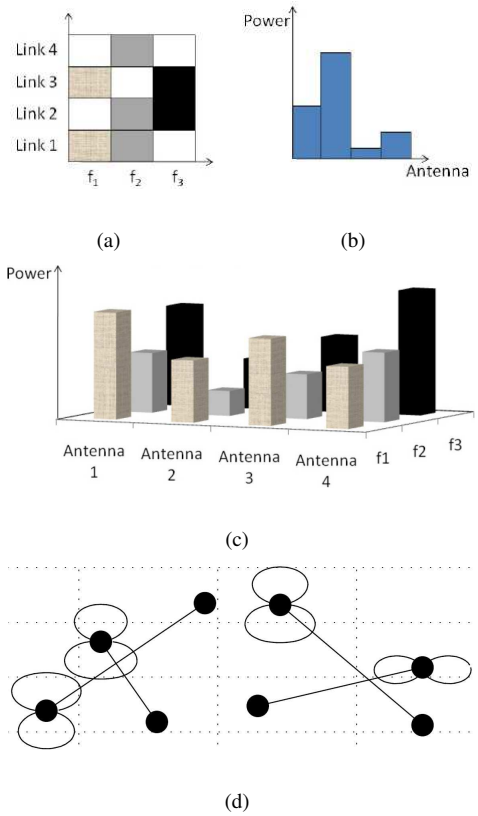


Fig. 1. Power allocation in frequency (a), space (b), or both (c); Four transmit radiation patterns steering away from nearby receivers (d).

results are also applicable to CR systems that adapt the CR transmit powers to the surrounding environment [13].

Because of the challenges associated with power and spectrum optimization, most existing works on MIMO CR systems (e.g., [14] [15] [11]) do not consider optimization over the frequency dimension. The extension of these works to multi-band MIMO CRNs is not trivial for two reasons. First, scalar-value algorithms (e.g., the bisection search in [11]) used for a single-band MIMO ad hoc network will not work when searching for optimal vectors in multi-band MIMO CRNs. Second, if one separately applies results from single-band MIMO to each individual band of a multi-band MIMO CR system, the achieved throughput is often low, as the resulting network operation points do not meet the optimal conditions (discussed later). In this paper, simulations are used to compare the performance of such approaches to that of our algorithms.

Motivated by the above, this paper develops low-complexity distributed algorithms that configure the transmit antenna radiation directions and allocate power over all data streams, specified by space (subindex s) and frequency (subindex k) dimensions, so that the MIMO CRN's throughput is maximized. We model the power and spectrum allocation in MIMO CRNs as a price-based noncooperative game [16]. To manage interference, we derive a diagonal block pricing-factor matrix. This matrix is user-dependent, and is used to capture the interference effect from a transmitter to unintended receivers. Hence, it is a function of the node's precoding matrices as well as its neighboring receivers. The pricing-factor matrix not only improves the Nash-Equilibrium (NE) of the game, but also drives the game towards a locally optimal point of the centralized problem. Exploiting the strong duality in convex optimization, we design a low-complexity distributed algorithm to determine the set of precoding matrices (*best reponse*) for each node. The dimensionality of the distributed algorithm is only $K+1$, where K is the number of frequency bands, i.e., it does not depend on the antenna array size. We also develop a centralized algorithm for the network optimization problem, where nodes are assumed to work in a cooperative way. Simulations show that the performance of the distributed algorithm is almost the same as that of the centralized one.

Throughout the paper, we use $(.)^*$ to denote the conjugate matrix, $(.)^H$ to denote the Hermitian matrix transpose, tr$(.)$ for the trace of a matrix, and $\det(.)$ for the determinant. Matrices and vectors are indicated in boldface.

In Section II, we present the network model and the problem formulation. The noncooperative game analysis, optimal pricing policy, convergence proof, and the distributed algorithm are given in Section III. The centralized algorithm is developed using augmented Lagrangian multipliers in Section IV. Numerical results are discussed in Section V. Concluding remarks are provided in Section VI.

II. Problem Formulation

We consider a CRN that coexists with several PR networks. The CRN consists of N links. Each CR node is equipped with M antennas. The spectrum to be allocated is comprised of K orthogonal frequency bands (also referred to as channels). Let $\Phi_N \stackrel{\text{def}}{=} \{1,2,\ldots,N\}$ and $\Psi_K \stackrel{\text{def}}{=} \{1,2,\ldots,K\}$ denote the sets of CR links and channels, respectively. Each CR user i can simultaneously communicate over multiple frequency bands, denoted by the set S_i. We impose a half-duplex constraint on all transmissions, meaning that a CR cannot transmit and receive at the same time.

The transmitter of each CR link can send up to M independent data streams on its M antennas over a given channel. A node controls the emitted antenna pattern and power allocation for these streams through its precoding matrices. Formally, for frequency band k, let $\mathbf{x}_i^{(k)}$ be a column vector of M information symbols, sent from node i to its destination node $d(i)$. Each element of $\mathbf{x}_i^{(k)}$ belongs to one data stream. Let $\tilde{\mathbf{T}}_i^{(k)}$ denote the precoding matrix of node i on channel k. Then, the actual transmit vector is $\tilde{\mathbf{T}}_i^{(k)}\mathbf{x}_i^{(k)}$.

We allow for spectrum sharing among different CR links. This assumption is in contrast to the case that restricts to one CR transmission on a given frequency channel in the same neighborhood. In our setup, several CR links can simultaneously occupy the same channel. Specifically, for a frequency band k, the received signal vector $\mathbf{y}_{d(i)}^{(k)}$ at the receiver $d(i)$ of link $(i,d(i))$ is given by:

$$\mathbf{y}_{d(i)}^{(k)} = \mathbf{H}_{d(i),i}^{(k)}\tilde{\mathbf{T}}_i^{(k)}\mathbf{x}_i^{(k)} + \sum_{j\in\Phi_N\setminus\{i\}} \mathbf{H}_{d(i),j}^{(k)}\tilde{\mathbf{T}}_j^{(k)}\mathbf{x}_j^{(k)} + \mathbf{N}_k \quad (1)$$

where the first term in the RHS of (1) is the desired signal sent from transmitter i and $\mathbf{H}_{d(i),i}^{(k)}$ is the channel gain matrix on channel k from the transmitter i to the receiver $d(i)$. Specifically, $\mathbf{H}_{d(i),i}^{(k)} \stackrel{\text{def}}{=} |\mathbf{h}_1\mathbf{h}_2\ldots\mathbf{h}_M|$, where $\mathbf{h}_s$ is an $M\times 1$ column vector of channel gains from transmit antenna s to all M receiving antennas, $s=1,\ldots,M$. We assume a flat-fading channel. Each entry of $\mathbf{H}_{d(i),i}^{(k)}$ is a complex Gaussian variable with zero mean and unit variance. The second term in the expression of $\mathbf{y}_{d(i)}^{(k)}$ represents interference from other CR links that share channel k with link $(i,d(i))$. $\mathbf{N}_k$ is an $M\times 1$ complex Gaussian noise vector with identity covariance matrix $\mathbf{I}$, representing the noise floor as well as interference from nearby PR users on band k.

The Shannon capacity of link $(i,d(i))$, referred to as (i) for short, on channel k is [3]:

$$R_{(i)}^{(k)} = \log\det(\mathbf{I} + \tilde{\mathbf{T}}_i^{(k)H}\mathbf{H}_{d(i),i}^{(k)H}{\mathbf{C}_{d(i)}^{(k)}}^{-1}\mathbf{H}_{d(i),i}^{(k)}\tilde{\mathbf{T}}_i^{(k)}) \quad (2)$$

where $\mathbf{C}_{d(i)}^{(k)}$ is the noise-plus-interference covariance matrix at $d(i)$ over band k, given by:

$$\mathbf{C}_{d(i)}^{(k)} = \mathbf{I} + \sum_{j \in \Phi_N \setminus \{i\}} \mathbf{H}_{d(i),j}^{(k)} \tilde{\mathbf{T}}_j^{(k)} \tilde{\mathbf{T}}_j^{(k)H} \mathbf{H}_{d(i),j}^{(k)H}.$$

The total channel rate over all frequency bands of link i is:

$$R_{(i)} = \sum_{k \in \Psi_K} R_{(i)}^{(k)}. \tag{3}$$

We use $P_{s,k}^{(i)}$ to denote the power allocated on band k (frequency dimension) at stream s (space dimension or antenna) of CR user i. $P_{s,k}^{(i)}$ is the entry (s,s) on the diagonal of matrix $(\tilde{\mathbf{T}}_i^{(k)} \tilde{\mathbf{T}}_i^{(k)H})$. For user i, the total power allocated on all frequency bands and all antennas should not exceed its maximum power budget $P_{\max}$ (we assume an identical power limit for all CR users). Consequently,

$$\sum_{k \in \Psi_K} \sum_{s=1}^{M} P_{s,k}^{(i)} = \sum_{k \in \Psi_K} \text{tr}(\tilde{\mathbf{T}}_i^{(k)} \tilde{\mathbf{T}}_i^{(k)H}) \leq P_{\max}. \tag{4}$$

Spectrum sharing between CR and PR transmissions takes two forms: *spectrum overlay* and *spectrum underlay*. In the former, CRs only occupy a channel if on that channel, no PR is detected (also known as a *detect-and-avoid* mechanism). Spectrum underlay allows CR users to occupy a channel even if PRs are detected, provided that the transmissions of CRs do not deteriorate the quality of service for PR users. There are two methods to realize underlay spectrum sharing: *static* and *dynamic*. The static method requires that the transmit power of CRs on frequency channel k is always less than a given power mask $P_{mask}(k)$. Let $\mathbf{P}_{mask} \stackrel{\text{def}}{=} (P_{mask}(1), P_{mask}(2), \ldots, P_{mask}(K))$ denote the power mask on all channels. Instead of specifying hard constraints on the transmit powers of CRs, the dynamic method adapts these transmit powers of CRs to activities from neighboring PRs and other CRs so that the total interference (from all CRs and PRs) at a nearby PR receiver does not exceed a threshold. Though the dynamic method may result in higher network throughput, it requires coordination among CRs and PRs and can only statistically guarantee the PRs' quality of service. This is due to the fact that it is impractical to accurately model and estimate the interference from CR and PR links. In this paper, we use the static method, implying that:

$$\sum_{s=1}^{M} P_{s,k}^{(i)} = \text{tr}(\tilde{\mathbf{T}}_i^{(k)} \tilde{\mathbf{T}}_i^{(k)H}) \leq P_{mask}(k). \tag{5}$$

It should be noted that our subsequent analysis is also applicable to the dynamic method and the detect-and-avoid mechanism.

We aim at maximizing the CRN throughput. Mathematically, the network optimization problem can be stated as follows:

$$\begin{aligned} &\underset{\{\tilde{\mathbf{T}}_i^{(k)}, \forall k \in \Psi_K, \forall i \in \Phi_N\}}{\text{maximize}} \quad \sum_{i \in \Phi_N} R_{(i)} \\ &\text{s.t.} \\ &\text{C1:} \quad \sum_{k \in \Psi_K} \text{tr}(\tilde{\mathbf{T}}_i^{(k)} \tilde{\mathbf{T}}_i^{(k)H}) \leq P_{\max}, \ \forall i \in \Phi_N \\ &\text{C2:} \quad \text{tr}(\tilde{\mathbf{T}}_i^{(k)} \tilde{\mathbf{T}}_i^{(k)H}) \leq P_{mask}(k), \ \forall k \in \Psi_K, \forall i \in \Phi_N. \end{aligned} \tag{6}$$

III. Game Theoretic Design

The network optimization problem (6) is not convex due to the presence of interference among CR users that share the same frequency band. Thus, even in a centralized manner, computing the globally optimal solution is prohibitively expensive. Thus, we reformulate it using game theory and derive a pricing function for each CR link that guarantees a locally optimal solution for problem (6), found in a distributed manner.

A. Game Formulation

A noncooperative game is characterized by its set of players, their action/strategy space, and their utility/payoff functions. For the underlying CRN, the set of CR links Φ_N represents the set of players. The action space is the union of the action spaces of various players, subject to constraints C1 and C2 in (6). The action/strategy space for each player is the set of all possible precoding matrices for the K frequency channels in Ψ_K. Formally, an action from the action space of link i is denoted by $\tilde{\mathbf{T}}_i \stackrel{\text{def}}{=} (\tilde{\mathbf{T}}_i^{\{1\}}, \tilde{\mathbf{T}}_i^{\{2\}}, \ldots, \tilde{\mathbf{T}}_i^{(k)})$, which can be viewed as an $M \times KM$ block matrix, comprised of K $M \times M$ matrices. Let $\tilde{\mathbf{T}}_{-i} \stackrel{\text{def}}{=} (\tilde{\mathbf{T}}_1, \tilde{\mathbf{T}}_2, \ldots, \tilde{\mathbf{T}}_{i-1}, \tilde{\mathbf{T}}_{i+1}, \ldots, \tilde{\mathbf{T}}_N)$ be the set of actions from all links, except link i. The utility or payoff of player i for its action $\tilde{\mathbf{T}}_i$ is mapped to link i's Shannon rate, which also depends on the selection of the precoding matrices from other CR links $\tilde{\mathbf{T}}_{-i}$:

$$\begin{aligned} &U_i(\tilde{\mathbf{T}}_i, \tilde{\mathbf{T}}_{-i}) \stackrel{\text{def}}{=} R_{(i)} \\ &= \sum_{k \in \Psi_K} \log\det(\mathbf{I} + \tilde{\mathbf{T}}_i^{(k)H} \mathbf{H}_{d(i),i}^{(k)H} {\mathbf{C}_{d(i)}^{(k)}}^{-1} \mathbf{H}_{d(i),i}^{(k)} \tilde{\mathbf{T}}_i^{(k)}). \end{aligned} \tag{7}$$

Due to the noncooperative nature of the game, the transmitter of each link allocates its transmission power over both space and frequency dimensions, and configures its radiation directions to maximize its own return. Formally, each CR user i solves the following problem for its precoding matrix set $\tilde{\mathbf{T}}_i$:

$$\begin{aligned} &\underset{\{\tilde{\mathbf{T}}_i^{(k)}, \forall k \in \Psi_K\}}{\text{maximize}} \quad U_i(\tilde{\mathbf{T}}_i, \tilde{\mathbf{T}}_{-i}) \\ &\text{s.t.} \\ &\text{C1':} \quad \sum_{k \in \Psi_K} \text{tr}(\tilde{\mathbf{T}}_i^{(k)} \tilde{\mathbf{T}}_i^{(k)H}) \leq P_{\max} \\ &\text{C2':} \quad \text{tr}(\tilde{\mathbf{T}}_i^{(k)} \tilde{\mathbf{T}}_i^{(k)H}) \leq P_{mask}(k), \ \forall k \in \Psi_K. \end{aligned} \tag{8}$$

By solving the above problem, CR users implicitly interact with each other through their choice of the precoding matrices. Under some conditions, the game reaches a NE where no CR user has incentive to unilaterally deviate from. However, as each CR user behaves selfishly, the resulting NE is often far from the Pareto optimum, and network throughput can be low.

To drive the above noncooperative game to a better NE, i.e., achieve higher social welfare, we use a pricing or taxation mechanism to encourage selfish players to work in a cooperative manner [17]. Pricing makes players more socially responsible for their actions. The utility function with price is defined as follows:

$$U_i^{'}(\tilde{\mathbf{T}}_i, \tilde{\mathbf{T}}_{-i}) \stackrel{\text{def}}{=} U_i(\tilde{\mathbf{T}}_i, \tilde{\mathbf{T}}_{-i}) - F_u(\tilde{\mathbf{T}}_i) \tag{9}$$

where $F_u(\tilde{\mathbf{T}}_i)$ is the pricing function for link i. Consequently, we come up with the following noncooperative game with pricing:

$$\begin{aligned} &\underset{\{\tilde{\mathbf{T}}_i^{(k)} \forall k \in \Psi_K\}}{\text{maximize}} \quad U_i^{'}(\tilde{\mathbf{T}}_i, \tilde{\mathbf{T}}_{-i}), \ \ \forall i \in \Phi_N \\ &\text{s.t.} \\ &\text{C1' and C2' as in problem (8).} \end{aligned} \tag{10}$$

B. Pricing Policy

A Pareto-optimal pricing policy is one that drives the game to a NE on the Pareto frontier. An optimal pricing policy is one that yields the game to a NE that is identical to the globally optimal solution of the non-convex problem (6). However, deriving such a pricing function is often difficult for two reasons. First, it is hard to characterize the optimal or the Pareto-optimal pricing policy, making it not possible to quantify

the performance gap between these policies and the achieved NE. Second, an optimal pricing function that requires global network information is impractical for a distributed network. To improve the efficiency of the NE, the pricing functions in the literature are usually based on heuristics. For instance, the pricing functions in [18] are suboptimal linear functions with a fixed pricing-factor.

In economics, the pricing function can take various forms to account for various marketing and pricing policies, e.g., volume discount, coupon discount, etc. In the context of network resource allocation, both linear (e.g., [2] [19]) and nonlinear [1] pricing functions have been proposed. In this paper, we define the pricing function $F_u(\tilde{\mathbf{T}}_i)$ as follows:

$$F_u(\tilde{\mathbf{T}}_i) = \text{tr}\left[\tilde{\mathbf{T}}_i^H \times \mathbf{A}_i \times \tilde{\mathbf{T}}_i\right] \tag{11}$$

where

$$\mathbf{A}_i = \begin{bmatrix} \mathbf{A}_i^{(1)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_i^{(2)} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_i^{(K)} \end{bmatrix} \tag{12}$$

is an $KM \times KM$ block diagonal matrix, consisting of K blocks along its diagonal. The kth block $\mathbf{A}_i^{(k)}$ is an $M \times M$ positive-semidefinite matrix. $\mathbf{A}_i$ is referred to as the pricing-factor matrix of CR link i and $\mathbf{A}_i^{(k)}$ is referred to as the pricing-factor matrix at frequency band k of link i. The following theorem guarantees the existence of a NE of the game (10).

Theorem 1: There exists at least one NE for the noncooperative game in (10).

Proof: See [20]. □

The above game can have more than one NE. To guarantee a lower bound on the efficiency of the achieved NE, we propose in the next theorem a user-dependent pricing function. The proposed pricing policy ensures that at the resulting NE, the CRN throughput is at least as good as that of a locally optimal solution to the network optimization problem (6).

Theorem 2: For the game in (10) to converge to a NE at which the CRN's throughput equals to that of a locally optimal solution of problem (6), the pricing-factor matrix $\mathbf{A}_i$ in (12) must have its k block matrix $\mathbf{A}_i^{(k)}$ of the following form:

$$\mathbf{A}_i^{(k)} = \sum_{j \in \Phi_N \setminus \{i\}} \mathbf{H}_{d(j),i}^{(k)H} {\mathbf{C}_{d(j)}^{(k)}}^{-1} \mathbf{H}_{d(j),j}^{(k)} [(\tilde{\mathbf{T}}_j^{(k)} \tilde{\mathbf{T}}_j^{(k)H})^{-1} + \mathbf{H}_{d(j),j}^{(k)H} {\mathbf{C}_{d(j)}^{(k)}}^{-1} \mathbf{H}_{d(j),j}^{(k)}]^{-1} \mathbf{H}_{d(j),j}^{(k)H} {\mathbf{C}_{d(j)}^{(k)}}^{-1} \mathbf{H}_{d(j),i}^{(k)} \tag{13}$$

Proof: See Appendix I. □

To compute the pricing-factor matrix $\mathbf{A}_i$ in (12), a CR transmitter i needs to obtain feedback regarding the interference-plus-noise covariance, the precoding, and the channel matrices from all other links. In practice, if the channel gain matrix from i to $d(j)$ is too weak, i.e., $\mathbf{H}_{d(j),i}^{(k)} \approx \mathbf{0}$, there is no need for $d(j)$ to send its feedback to i. Hence, i only gets feedback from receivers $d(j)$ that are within i's vicinity. It is also worth noting that the feedback information is locally available at a receiver $d(j)$ as a byproduct of its decoding process (i.e., successive interference cancelation (SIC) receivers [3]). The kth block $\mathbf{A}_i^{(k)}$ of the pricing factor matrix in (13) agrees with that in [11] for a single-band MIMO ad hoc network using first-order Taylor series approximation. How a MAC protocol design can support the computation of the above pricing-factor matrix at a node will be discussed shortly.

C. Best Response: Optimal Antenna Radiation Directions and Power Allocation

We now solve the individual utility optimization problem (10), from which a CR user finds its best response given the actions of other CR links. Noting that problem (10) is convex and that the Slater's conditions can easily be shown to hold [21], strong duality holds for problem (10), i.e., an optimal solution $\tilde{\mathbf{T}}_i$ to (10) should also solve the following dual problem (as in the case of a single frequency MIMO network [11]):

$$\text{DP}: \underset{\{\alpha_i^{(k)}, \gamma_i \geq 0, \forall k \in \Psi_K\}}{\text{minimize}} \quad D(\alpha_i^{(k)}, \gamma_i) \tag{14}$$

where $D(\alpha_i^{(k)}, \gamma_i)$ is the dual function, defined as:

$$D(\alpha_i^{(k)}, \gamma_i) = \underset{\{\tilde{\mathbf{T}}_i^{(k)}, \forall k \in \Psi_K\}}{\text{maximize}} \quad L_i(\tilde{\mathbf{T}}_i, \alpha_i^{(k)}, \gamma_i). \tag{15}$$

with $L_i(\tilde{\mathbf{T}}_i, \alpha_i^{(k)}, \gamma_i)$ is the Lagrangian function defined in (29).

The optimal matrix $\tilde{\mathbf{T}}_i$ of (10) is featured in the following theorem.

Theorem 3: The $M \times KM$ block matrix $\tilde{\mathbf{T}}_i$ that solves the individual utility optimization problem (or the user's best response) must have its kth block, the matrix $\tilde{\mathbf{T}}_i^{(k)}$, in a form of the generalized eigen matrix of the matrix $\mathbf{H}_{d(i),i}^{(k)H} {\mathbf{C}_{d(i)}^{(k)}}^{-1} \mathbf{H}_{d(i),i}^{(k)}$ and matrix $\mathbf{A}_i^{(k)} + (\alpha_i^{(k)} + \gamma_i)\mathbf{I}$, where $\alpha_i^{(k)}$ and γ_i are the optimal Lagrange multipliers of (10). In other words, the following equations must hold $\forall k \in \Psi_K$:

$$\mathbf{H}_{d(i),i}^{(k)H} {\mathbf{C}_{d(i)}^{(k)}}^{-1} \mathbf{H}_{d(i),i}^{(k)} \tilde{\mathbf{T}}_i^{(k)} = [\mathbf{A}_i^{(k)} + (\alpha_i^{(k)} + \gamma_i)\mathbf{I}] \tilde{\mathbf{T}}_i^{(k)} \mathbf{\Lambda}_i^{(k)} \tag{16}$$

where $\mathbf{\Lambda}_i^{(k)}$ is a given $M \times M$ diagonal matrix.

Proof: See [20]. □

As previously discussed, the precoding matrix $\tilde{\mathbf{T}}_i^{(k)}$ determines both the directions of the antenna radiation as well as how node i allocates its transmission power on different antennas over frequency band k. Theorem (3) states a class of matrices that the solution of (10) must belong to. This class provides the directions that a user i should point its antenna radiation to. The next step is to find the optimal power allocation $P_{s,k}^{(i)}$ for the set of KM data streams. To ensure that $\tilde{\mathbf{T}}_i^{(k)}$ belongs to the class of matrices specified by Theorem (3), we let:

$$\tilde{\mathbf{T}}_i^{(k)} = \mathbf{T}_i^{(k)} \times {\mathbf{P}_k^{(i)}}^{1/2} \tag{17}$$

where $\mathbf{T}_i^{(k)}$ is an $M \times M$ matrix with unit-norm column vectors that satisfies (16). This matrix can be found by normalizing the generalized eigen matrix $\tilde{\mathbf{T}}_i^{(k)}$. ${\mathbf{P}_k^{(i)}}^{1/2}$ is a square root matrix of the $M \times M$ diagonal matrix $\mathbf{P}_k^{(i)}$ whose diagonal entry (s,s) is the power allocated to sub-channel (s,k), $P_{s,k}^{(i)}$. We can verify that the expression of $\tilde{\mathbf{T}}_i^{(k)}$ in (17) satisfies (16). As $\tilde{\mathbf{T}}_i^{(k)}$ is a generalized eigen matrix of the matrix $\mathbf{H}_{d(i),i}^{(k)H} {\mathbf{C}_{d(i)}^{(k)}}^{-1} \mathbf{H}_{d(i),i}^{(k)}$ and matrix $\mathbf{A}_i^{(k)} + (\alpha_i^{(k)} + \gamma_i)\mathbf{I}$, then $\mathbf{T}_i^{(k)}$ must also be an eigen matrix to each of the two matrices [22]. Thus, the following equations hold:

$$\mathbf{T}_i^{(k)H} [\mathbf{H}_{d(i),i}^{(k)H} {\mathbf{C}_{d(i)}^{(k)}}^{-1} \mathbf{H}_{d(i),i}^{(k)}] \mathbf{T}_i^{(k)} = \mathbf{\Pi}_i^{(k)} \tag{18}$$

$$\mathbf{T}_i^{(k)H} [\mathbf{A}_i^{(k)} + (\alpha_i^{(k)} + \gamma_i)\mathbf{I}] \mathbf{T}_i^{(k)} = \mathbf{\Omega}_i^{(k)} \tag{19}$$

where $\mathbf{\Pi}_i^{(k)}$ and $\mathbf{\Omega}_i^{(k)}$ are $M \times M$ (diagonal) matrices of the generalized eigenvalues of the matrices $\mathbf{H}_{d(i),i}^{(k)H} {\mathbf{C}_{d(i)}^{(k)}}^{-1} \mathbf{H}_{d(i),i}^{(k)}$ and $\mathbf{A}_i^{(k)} + (\alpha_i^{(k)} + \gamma_i)\mathbf{I}$, respectively.

Plugging (18) and (19) into the Lagrangian function (29), we have:

$$L_i(\tilde{\mathbf{T}}_i, \alpha_i^{(k)}, \gamma_i) = \sum_{k\in\Psi_K}\{\sum_{s=1}^{M}\{\log(1+P_{s,k}^{(i)}\text{diag}_s(\mathbf{\Pi}_i^k)) - P_{s,k}^{(i)}\text{diag}_s(\mathbf{\Omega}_i^{(k)})\} + \alpha_i^{(k)}P_{mask}(k) + \frac{\gamma_i}{K}P_{\max}\}. \quad (20)$$

The optimal power allocation $P_{s,k}^{(i)}$ is obtained by equating the derivative of the Lagrangian (20) w.r.t $P_{s,k}^{(i)}$ to zero:

$$\frac{\partial L_i(\tilde{\mathbf{T}}_i, \alpha_i^{(k)}, \gamma_i)}{\partial P_{s,k}^{(i)}} = \frac{\text{diag}_s(\mathbf{\Pi}_i^k)}{1+P_{s,k}^{(i)}\text{diag}_s(\mathbf{\Pi}_i^k)} - \text{diag}_s(\mathbf{\Omega}_i^{(k)}) = 0 \quad (21)$$

Thus:

$$P_{s,k}^{(i)} = \max\left(0, \frac{\text{diag}_s(\mathbf{\Pi}_i^{(k)}) - \text{diag}_s(\mathbf{\Omega}_i^{(k)})}{\text{diag}_s(\mathbf{\Pi}_i^{(k)})\text{diag}_s(\mathbf{\Omega}_i^{(k)})}\right) \quad (22)$$

Plugging (22) into (20), we obtain the dual function:

$$D(\alpha_i^{(k)}, \gamma_i) = \sum_{k\in\Psi_K}\{\sum_{s=1}^{M}\{\log\frac{\text{diag}_s(\mathbf{\Pi}_i^{(k)})}{\text{diag}_s(\mathbf{\Omega}_i^{(k)})} - 1 + \frac{\text{diag}_s(\mathbf{\Omega}_i^{(k)})}{\text{diag}_s(\mathbf{\Pi}_i^{(k)})}\} + \alpha_i^{(k)}P_{mask}(k) + \frac{\gamma_i}{K}P_{\max}\}$$
$$\forall s,k \text{ such that } \text{diag}_s(\mathbf{\Pi}_i^{(k)}) > \text{diag}_s(\mathbf{\Omega}_i^{(k)}) > 0. \quad (23)$$

To solve the DP (14) for $\alpha_i^{(k)}$ and γ_i, we note that the problem is convex, hence, any stationary point is a globally optimal solution. Moreover, as the objective function and constraints of the primal problem (10) is continuous w.r.t every entry of $\tilde{\mathbf{T}}_i$, the dual function $D(\alpha_i^{(k)}, \gamma_i)$ is differentiable w.r.t $\alpha_i^{(k)}$ and γ_i [23]. Hence, a gradient algorithm can be used to obtain the optimal Lagrangian multipliers $\alpha_i^{(k)}$ and γ_i by searching for a stationary point of the augmented Lagrangian of DP. It should be noted that even if the dual function is not differentiable (i.e., multiple subgradients may exist), a subgradient-based search algorithm with appropriate step size can still be used to converge to the optimal point [23]. The augmented Lagrangian of DP is given by:

$$L(\alpha_i^{(k)}, \gamma_i, p, \lambda^{(k)}) = D(\alpha_i^{(k)}, \gamma_i) + \frac{p}{2}\{(\max\{0, \lambda^{(1)} - p\gamma_i\})^2 - (\lambda^{(1)})^2\} + \frac{p}{2}\sum_{k\in\Psi_K}\{(\max\{0, \lambda^{(k+1)} - p\alpha_i^{(k)}\})^2 - (\lambda^{(k+1)})^2\} \quad (24)$$

where p is a positive penalty parameter (for violating the constraints) and $\lambda^{(k)}$'s are nonnegative Lagrangian multipliers.

Our gradient algorithm uses Armijo step with steepest descent direction. This search mechanism together with the above analysis are summarized in Algorithm 1. We emphasize that by exploiting the strong duality, this algorithm needs only to deal with $K+1$ variables, instead of $2KM^2$ variables of the primal problem (10).

Although the individual optimization (10) is to be solved distributedly at each node, at the achieved NE, network throughput is analytically guaranteed to be as good as that of a locally optimal point of the network optimization problem (6). Before developing a centralized algorithm that serves as a simulation's performance benchmark, let's briefly discuss how a MAC protocol can implement Algorithm 1 and its convergence behavior.

Algorithm 1 Distributed Algorithm for the Power Allocation and Spectrum Management Game

Input:
$\tilde{\mathbf{T}}_{-i} = [\tilde{\mathbf{T}}_1(t+1), ..., \tilde{\mathbf{T}}_{i-1}(t+1), \tilde{\mathbf{T}}_{i+1}(t), ..., \tilde{\mathbf{T}}_N(t)]$ with Gauss-Seidel iteration
$\tilde{\mathbf{T}}_{-i} = [\tilde{\mathbf{T}}_1(t), ..., \tilde{\mathbf{T}}_{i-1}(t), \tilde{\mathbf{T}}_{i+1}(t), ..., \tilde{\mathbf{T}}_N(t)]$ with Jacobi iteration
Initialize
$\tilde{\mathbf{T}}_i^{(k)}(t+1) \leftarrow \tilde{\mathbf{T}}_{\mathbf{i}}^{(\mathbf{k})}(\mathbf{t}), \gamma_i \leftarrow 0; \alpha_i^{(k)} \leftarrow 0, \forall k \in \Psi_K$
while $true$ **do**
  $\beta \leftarrow .7, \sigma \leftarrow .1$%*used in Armijo search*
  $\lambda^{(k)} \leftarrow .1 \forall k = 1 \dots (K+1)$
  $p \leftarrow 1$
  while $\partial L(\alpha_i^{(k)}, \gamma_i, p, \lambda^{(k)}) \neq 0$ **do**
    $step \leftarrow 0.1$
    $D \leftarrow \partial L(\alpha_i^{(k)}, \gamma_i, p, \lambda^{(k)})$
    $d \leftarrow -step \times D; m \leftarrow 0$
    {Find Armijo step size}
    while $L(\alpha_i^{(k)}, \gamma_i, p, \lambda^{(k)}) - L(\alpha_i^{(k)}+d, \gamma_i+d, p, \lambda^{(k)}) \leq -\sigma\beta^m step \partial L D$ **do**
      $step \leftarrow step \times \beta; m \leftarrow m+1$
      $d \leftarrow -step \times D$
    end while
    $\alpha_i^{(k)} \leftarrow \alpha_i^{(k)} + d, \gamma_i \leftarrow \gamma_i + d$
  end while
  if $\min(\alpha_i^{(k)}, \gamma_i, \forall k \in \Psi_K) \geq 0$ **break**
  $\forall k \in \Psi_K$:
  $\lambda^{(k)} \leftarrow \lambda^{(k)} - p\alpha_i^{(k)}$ **if** $\lambda^{(k)} - p\alpha_i^{(k)} > 0$ **else** $\lambda^{(k)} = 0$
  $\lambda^{(1)} \leftarrow \lambda^{(1)} - p\gamma_i$ **if** $\lambda^{(1)} - p\gamma_i \geq 0$ **else** $\lambda^{(1)} = 0$
  $p \leftarrow p \times \mu$%$\mu \geq 1$, *increase cost of violation*
end while
Plug γ_i, $\alpha_i^{(k)}$ into (16) (Theorem 3) to find $\mathbf{T}_i^{(k)}$
$\mathbf{P}_k^{(i)}$ is found from (18), (19), (22). $\tilde{\mathbf{T}}_i^{(k)}$ is found from (17).
RETURN $\tilde{\mathbf{T}}_i^{(k)}(t+1), \forall k \in \Psi_K$ at time $(t+1)$

D. Overview of a MAC Protocol

Using either a dedicated control channel or some frequency hopping mechanisms to establish an initial dialogue, a MAC protocol that executes the distributed Algorithm 1 can be designed. This protocol divides the time axis into three windows: *Access window*, *training window*, and *data window*. The access window is dedicated to CR nodes that have data to send. These nodes first exchange some initial rendezvous packets (e.g., CTS and RTS). After this phase, several pairs of CR users communicate during the training window, whose purpose is to exchange/negotiate transmit strategies (precoding matrices). The signalling packets in either the access or transmit windows can also be used to embed training sequences to obtain channel gain matrices. The data window then follows with multiple data packets sent using negotiated transmission strategies. A representative game-based MAC protocol can be found in [2].

To reduce the feedback overhead, one may relax the time scale of recalculating the pricing-factor matrix. This presents a tradeoff between throughput and feedback freshness. Specifically, the pricing-factor matrix can be recomputed following every data packet (packet-based) or a flow of packets (flow-based) [24] [25].

An important issue for protocol designers is how to set the size of the training window. That depends on the convergence speed of the updating process. To ensure that the training window is not too long, the updating and negotiation processes must converge. During the training window, a node can use either Gauss-Seidel (sequential) or Jacobi (parallel) iterations to update its precoding matrices. Though we cannot prove the

convergence under the Jacobi iteration, simulations show that the distributed algorithm converges faster with Jacobi iterations than with Gauss-Seidel (less than nine iterations for about ten links in Figure 4). The convergence behavior under the Gauss-Seidel iteration is claimed in the following theorem.

Theorem 4: Under the sequential updating procedure (Gauss-Seidel), the distributed Algorithm 1 drives the game (10) to its NE.

Proof: For brevity, we give an outline of the proof, and refer the reader to [20] for the complete proof. To establish the convergence, one can find a Lyapunov-type function of the precoding matrices and show that the function is non-decreasing and upper-bounded (e.g., [26]). The converged point must be a NE, otherwise one user can still unilaterally improve its return $U_i^{'}(\tilde{\mathbf{T}}_i, \tilde{\mathbf{T}}_{-i})$ (which violates the convexity of the individual problem (10)). □

IV. Centralized Algorithm

From a game theoretic perspective, a centralized algorithm can be obtained by formulating the problem as a cooperative game, where a network operator somehow controls the behaviors of all players in order to maximize the network throughput (total payoff). In this section, we use the augmented Lagrangian multiplier method to derive such a centralized algorithm. We rewrite problem (6) as follows:

$$\begin{aligned}
&\underset{\{\tilde{\mathbf{T}}_i^{(k)}, \forall k \in \Psi_K, \forall i \in \Phi_N\}}{\text{minimize}} \quad -\sum_{i\in\Phi_N} R_{(i)} \\
&\text{s.t.} \\
&c_i = \sum_{k\in\Psi_K} \text{tr}(\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}) - P_{\max} \le 0 \ \ \forall i \in \Phi_N \\
&c_{k,i} = \text{tr}(\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}) - P_{mask}(k) \le 0 \ \ \forall k \in \Psi_K, \forall i \in \Phi_N
\end{aligned} \tag{25}$$

The augmented Lagrangian of (25) is given by [23]:

$$\begin{aligned}
&L(\tilde{\mathbf{T}}, \alpha_i^{(k)}, \gamma_i, p) \\
&= -\sum_{i\in\Phi_N} R_{(i)} + \frac{p}{2}\sum_{i\in\Phi_N}\{(\max\{0, \gamma_i + pc_i\})^2 - (\gamma_i)^2\} \\
&\quad + \frac{p}{2}\sum_{i\in\Phi_N}\sum_{k\in\Psi_K}\{(\max\{0, \alpha_i^{(k)} + pc_{k,i}\})^2 - (\alpha_i^{(k)})^2\}
\end{aligned} \tag{26}$$

where p is a positive penalty parameter (for violating the constraints), and $\alpha_i^{(k)}$ and γ_i are nonnegative Lagrangian multipliers.

At a locally optimal solution, we have:

$$\begin{aligned}
0 = \frac{\partial L(\tilde{\mathbf{T}}, \alpha_i^{(k)}, \gamma_i, p)}{\partial \tilde{\mathbf{T}}_i^{(k)*}} &= -\sum_{j\in\Phi_N\setminus\{i\}} \frac{\partial R_j^{(k)}}{\partial \tilde{\mathbf{T}}_i^{(k)*}} - \frac{\partial R_i^{(k)}}{\partial \tilde{\mathbf{T}}_i^{(k)*}} \\
+ \frac{p}{2}\{\frac{\partial\{(\max\{0, \gamma_i + pc_i\})^2\}}{\partial \tilde{\mathbf{T}}_i^{(k)*}} &+ \frac{\partial\{(\max\{0, \alpha_i^{(k)} + pc_{k,i}\})^2\}}{\partial \tilde{\mathbf{T}}_i^{(k)*}}\}
\end{aligned} \tag{27}$$

The first term in (27) is computed in (34). Its second term is given as:

$$\begin{aligned}
&\frac{\partial R_i^{(k)}}{\partial \tilde{\mathbf{T}}_i^{(k)*}} \\
&= \mathbf{H}_{d(i),i}^{(k)H}(\mathbf{C}_{d(i)}^{(k)} + \mathbf{H}_{d(i),i}^{(k)}\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}\mathbf{H}_{d(i),i}^{(k)H})^{-1}\mathbf{H}_{d(i),i}^{(k)}\tilde{\mathbf{T}}_i^{(k)}.
\end{aligned} \tag{28}$$

Since c_i and $c_{k,i}$ are continuously differentiable w.r.t every entry of $\tilde{\mathbf{T}}$, the third and fourth terms in (27) are also continuously differentiable [23]. Their derivatives are as follows:

$$\frac{\partial\{(\max\{0, \gamma_i + pc_i\})^2\}}{\partial \tilde{\mathbf{T}}_i^{(k)*}} = \begin{cases} 0 & \text{if } \gamma_i + pc_i \le 0 \\ 2p(\gamma_i + pc_i)\tilde{\mathbf{T}}_i^{(k)} \end{cases}$$

$$\frac{\partial\{(\max\{0, \alpha_i^{(k)} + pc_{k,i}\})^2\}}{\partial \tilde{\mathbf{T}}_i^{(k)*}}\} = \begin{cases} 0 & \text{if } \alpha_i^{(k)} + pc_{k,i} \le 0 \\ 2p(\alpha_i^{(k)} + pc_{k,i})\tilde{\mathbf{T}}_i^{(k)} \end{cases}$$

As mentioned earlier, because the network optimization problem is not convex, the centralized algorithm can only lead to a locally optimal point. For that purpose, we use the gradient search algorithm with Armijo step size [23] to find $(\tilde{\mathbf{T}}, \alpha_i^{(k)}, \gamma_i, p)$ such that equation (27) holds for all frequency bands k and all users i. The details of the centralized algorithm is presented in Algorithm 2.

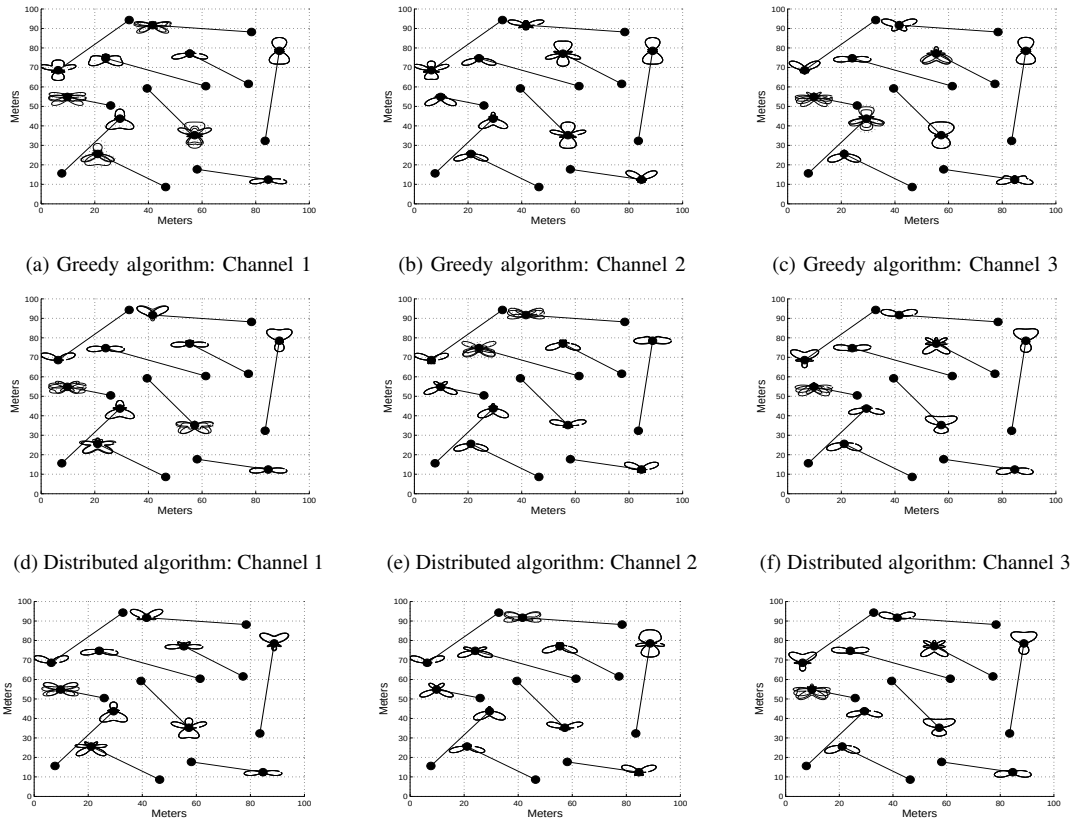
We emphasize that network throughput may vary from a locally optimal point to another. To account for such phenomenon, one can run the simulations multiple times with various starting points (initializations) and take the average of the achieved throughput. The running time of Algorithm 2 can be high, as it involves NKM^2 complex variables (or $2NKM^2$ real ones). To implement Algorithm 2, one should convert complex matrices to vectors of real variables.

Algorithm 2 Centralized Algorithm for the Social Optimization Problem (6)

Initialize
$\tilde{\mathbf{T}}_i^{(k)} \leftarrow \mathbf{I}, \gamma_i \leftarrow 0; \alpha_i^{(k)} \leftarrow 0, \forall k \in \Psi_K, \forall i \in \Phi_N$
while $true$ **do**
  $\beta \leftarrow .7, \sigma \leftarrow .1$*%used in Armijo search*
  $\gamma_i \leftarrow 0, \alpha_i^{(k)} \leftarrow 0, \forall k \in \Psi_K, \forall i \in \Phi_N$
  $p \leftarrow 1$
  while $\partial L(\tilde{\mathbf{T}}, \alpha_i^{(k)}, \gamma_i, p) \neq 0$ **do**
    $step \leftarrow 0.1$
    $D \leftarrow \partial L(\tilde{\mathbf{T}}, \alpha_i^{(k)}, \gamma_i, p)$
    $d \leftarrow -step \times D; m \leftarrow 0$
    {Find Armijo step size}
    while $L(\tilde{\mathbf{T}}, \alpha_i^{(k)}, \gamma_i, p) - L(\tilde{\mathbf{T}} + d, \alpha_i^{(k)}, \gamma_i, p) \le -\sigma\beta^m step\partial L D$ **do**
      $step \leftarrow step \times \beta; m \leftarrow m + 1$
      $d \leftarrow -step \times D$
    end while
    $\tilde{\mathbf{T}} \leftarrow \tilde{\mathbf{T}} + d$
  end while
  if $\max(c_i, c_{i,k}, \forall k \in \Psi_K, \forall i \in \Phi_N) \le 0$ **break**
  $\forall k \in \Psi_K, \forall i \in \Phi_N$:
  $\gamma_i = \gamma_i + pc_i$ **if** $\gamma_i + pc_i \ge 0$ **else** $\gamma_i = 0$
  $\alpha_i^{(k)} = \alpha_i^{(k)} + pc_{k,i}$ **if** $\alpha_i^{(k)} + pc_{k,i} \ge 0$ **else** $\alpha_i^{(k)} = 0$
  $p \leftarrow p \times \mu$*%*$\mu \ge 1$*, increase cost of violation*
end while
RETURN $\tilde{\mathbf{T}}_i^{(k)}$, $\forall k \in \Psi_K, \forall i \in \Phi_N$

V. Numerical Results

In this section, we numerically evaluate the performance of the distributed algorithm using MATLAB-based simulations. We compare the network throughput of the distributed algorithm with the centralized one and with a greedy algorithm, in which nodes selfishly attempt to maximize their own rates. The greedy algorithm is exactly the same as the distributed one except that its pricing-factor matrix $\mathbf{A}_i$ is a null matrix. Another algorithm called *uniform* is obtained by uniformly dividing a node's total transmit power over all available channels and then applying the single-band approach in [11] for each channel. We emphasize that this uniform algorithm does not meet the optimality conditions (32) of the network problem (6).



(a) Greedy algorithm: Channel 1 (b) Greedy algorithm: Channel 2 (c) Greedy algorithm: Channel 3

(d) Distributed algorithm: Channel 1 (e) Distributed algorithm: Channel 2 (f) Distributed algorithm: Channel 3

(g) Centralized algorithm: Channel 1 (h) Centralized algorithm: Channel 2 (i) Centralized algorithm: Channel 3

Fig. 2. Antenna radiation patterns under the greedy, distributed, and centralized algorithms.

Because the number of variables in the centralized algorithm is quite high ($2NKM^2$), its running time can be very long. Therefore, to compare the performance of the four algorithms, we consider a CRN of $N = 10$ links with $K = 3$ bands. The carrier frequencies for these bands are $f_1 = 2.4$ GHz, $f_2 = 2.7$ GHz, and $f_3 = 3$ GHz. These bands have identical channel bandwidth of 1 MHz. The antenna array size $M = 4$. The results are averaged over 30 runs. In each run, N links are randomly placed in a 100 meter $\times$ 100 meter square. The maximum power at each node is $2W$ and the power mask is $0.8W$ on all frequency bands. The channel fading is flat with free-space attenuation factor of 2. The spreading angles of the signal at the receive antennas range from $-\pi/5$ to $\pi/5$. For the lowest frequency, we assume that the received power at a reference distance of 100 meters reduces 10 dB compared with the transmit power. To account for the frequency-dependent attenuation factor, we assume that the received power at the reference distance decreases 2 dB more if the frequency increases by 300 MHz. As mentioned before, the noise from PR transmissions is treated as floor noise that together with the thermal noise are normalized to a unit variance. The initializations of the precoding matrices are different for all algorithms.

A snapshot of the network topology and antenna radiation patterns (at the converged points) over different frequencies is shown in Figure 2. We can visually note that the transmitters under the distributed and centralized algorithms often steer their beams away from neighboring receivers. This results from attempting to minimize the price function (11). It can also be seen that the antenna patterns of the distributed and centralized algorithms are very similar, suggesting the two algorithms may converge to the same point.

Figure 3 depicts the network throughput under four algorithms (distributed, centralized, greedy, and uniform) versus the number of iterations. Though the network performance at the converged points for the distributed and centralized algorithms change with their starting points, after averaging over multiple runs with different initializations, the throughput of the distributed algorithm is almost the same as that of the centralized one. We also notice that by using the proposed pricing policy to regulate interference, the distributed algorithm almost doubles the network throughput compared with the greedy algorithm. The uniform algorithm also improves network throughput over the greedy one but it remains inferior to the distributed algorithm. This is because the uniform algorithm evenly allocates its power over all available channels and does not do optimize over the frequency dimension, while the distributed algorithm attempts to optimize the antenna radiation patterns and the power allocation over both space and frequency.

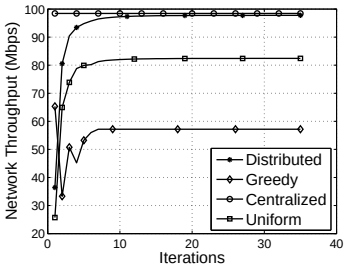


Fig. 3. Network throughput vs. iterations.

To evaluate the energy efficiency of the four algorithms, we record the average power consumption and power allocation over all nodes and all run. As shown in Table I, without regulating interference, nodes under the greedy algorithm selfishly compete for their own throughput by always using their maximum power ($2W$), leading to the highest power consumption among the four algorithms. The power consumption for the distributed algorithm is comparable to that of the centralized and uniform algorithms, and 10% less than that of the greedy one. Power allocation over both space and frequency at a

representative node under the distributed algorithm is shown in Table II. From Tables I and II, we notice that the inequality constraints in problems (10) and (6) are not active at their solutions. That is because transmitting at high power may be expensive due to the proposed pricing method.

Channels	Centralized	Greedy	Distributed	Uniform
f_1	0.768	0.71	0.76	0.658
f_2	0.643	0.66	0.61	0.556
f_3	0.422	0.63	0.44	0.627
Total (W)	1.823	2.00	1.81	1.831

TABLE I

AVERAGE POWER CONSUMPTION AND POWER ALLOCATION OVER DIFFERENT CHANNELS (IN WATTS).

Antennas	f_1	f_2	f_3
1	0.135	0.085	$0.15e-10$
2	0.209	0.386	0.02
3	0.550	0.314	$0.06e-10$
4	0.194	0.035	0.305
Total=1.913(W)	0.788	0.8	0.325

TABLE II

POWER ALLOCATION AT A NODE OVER SPACE AND FREQUENCY DIMENSIONS UNDER THE DISTRIBUTED ALGORITHM (IN WATTS).

We say that the algorithm converges if the change in the throughput of one iteration (relative to the previous iteration) is less than a given threshold (i.e., 3%). The convergence speed of the distributed algorithm versus the number of links is shown in Figure 4. As we can see, under both updating procedures (Jacobi and Gauss-Seidel), the distributed algorithm converges very fast. Using Jacobi iteration results in faster convergence.

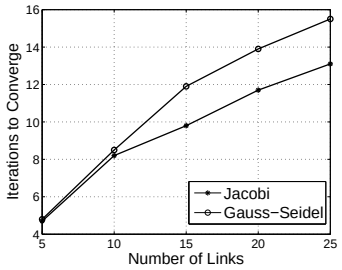


Fig. 4. Convergence speed of the distributed algorithm.

Figure 5 depicts the network throughput under the distributed and greedy algorithms versus the number of links using Jacobi iteration. The distributed algorithm consistently improves the throughput over the greedy one. The improvement becomes more significant with a higher number of links. That is because, as node density increases (higher number of links), network interference increases. Interference management becomes more critical and has more impact on throughput.

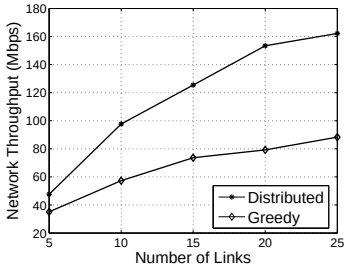


Fig. 5. Network throughput vs. the number of links.

VI. CONCLUSIONS

In this work, we investigated the spectrum sharing problem in multi-antenna CRNs. By adjusting the precoding matrices, we allocate power over both the frequency and space dimensions while managing the antenna's radiation beams to reduce network interference, aiming at maximizing the network throughput. Using game theory and the strong duality in convex optimization, we designed a low-complexity distributed algorithm that achieves the same throughput as a locally optimal point of the non-convex centralized network problem. The key idea behind the algorithm is the introduction of a diagonal block pricing-factor matrix for each CR. This matrix regulates network interference by encouraging CRs to work in a cooperative manner. Simulations showed that the proposed algorithm dramatically improves network throughput and achieves higher energy efficiency, compared with other solutions. We believe that controlling radiation beams in multi-antenna systems is an important optimization dimension, besides controlling streams and powers.

REFERENCES

[1] W. Wang, Y. Cui, T. Peng, and W. Wang, "Noncooperative power control game with exponential pricing for cognitive radio network," in *Proceedings of the IEEE 65th Vehicular Technology Conference*, April 2007.

[2] F. Wang, M. Krunz, and S. Cui, "Price-based spectrum management in cognitive radio networks," *IEEE Journal of Selected Topics in Signal Processing,*, vol. 2, no. 1, pp. 74–87, 2008.

[3] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, May 2005.

[4] K. Sundaresan, R. Sivakumar, M. Ingram, and T.-Y. Chang, "A fair medium access control protocol for ad-hoc networks with MIMO links," in *Proceedings of the INFOCOM Conference*, vol. 4, March 2004, pp. 2559–2570.

[5] K. Sundaresan and R. Sivakumar, "Routing in ad-hoc networks with MIMO links," in *Proceedings of the ICNP Conference*, Nov. 2005.

[6] S. Chu and X. Wang, "Opportunistic and cooperative spatial multiplexing in MIMO ad hoc networks," in *Proceedings of MOBIHOC Conference*, 2008, pp. 63–72.

[7] J. Liu, Y. Hou, Y. Shi, and H. Sherali, "On performance optimization for multi-carrier MIMO ad hoc networks," in *Proceedings of the ACM MOBIHOC Conference*, May 2009, pp. 43–54.

[8] U. Phuyal, A. Punchihewa, V. Bhargava, and C. Despins, "Power loading for multicarrier cognitive radio with MIMO antennas," in *Proceedings of the IEEE Wireless Communications and Networking Conference (WCNC)*, April 2009, pp. 1–5.

[9] Y. J. Zhang and A. So, "Optimal spectrum sharing in MIMO cognitive radio networks via semidefinite programming," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 2, pp. 362–373, Feb. 2011.

[10] R. Zhang and Y.-C. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 88–102, Feb. 2008.

[11] S.-J. Kim and G. B. Giannakis, "Optimal resource allocation for MIMO ad hoc cognitive radio networks," *IEEE Transactions on Information Theory*, vol. 57, no. 5, pp. 3117–3131, May 2011.

[12] Z. Tian, G. Leus, and V. Lottici, "Joint dynamic resource allocation and waveform adaptation for cognitive networks," *IEEE Journal on Selected Areas in Communications,*, vol. 29, no. 2, pp. 443–454, Feb. 2011.

[13] H. Salameh, M. Krunz, and O. Younis, "MAC protocol for opportunistic cognitive radio networks with soft guarantees," *IEEE Transactions on Mobile Computing*, vol. 8, no. 10, pp. 1339–1352, Oct. 2009.

[14] J. Wang, G. Scutari, and D. Palomar, "Robust MIMO cognitive radio via game theory," *IEEE Transactions on Signal Processing*, vol. 59, no. 3, pp. 1183–1201, March 2011.

[15] G. Scutari and D. Palomar, "MIMO cognitive radio: a game theoretical approach," *IEEE Transactions on Signal Processing*, vol. 58, no. 2, pp. 761–780, Feb. 2010.

[16] M. J. Osborne, *An Introduction to Game Theory*. Oxford University Press, 2004.

[17] J. Hirshleifer, A. Glazer, and D. Hirshleifer, *Price Theory and Applications Decisions, Markets, and Information*. Cambridge University Press, 2005.

[18] C. Saraydar, N. Mandayam, and D. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Transactions on Communications*, vol. 50, no. 2, pp. 291–303, Feb. 2002.

[19] W. Yu, "Competition and cooperation in multi-user communication environments," *PhD Thesis, Stanford University, Standford, CA*, 2002.

[20] D. Nguyen and M. Krunz, "Spectrum management and power allocation in MIMO cognitive networks," University of Arizona, Tech. Rep. TR-UA-ECE-2011-2, August 2011. [Online]. Available: http://www.ece.arizona.edu/~krunz

[21] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[22] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 1990.

[23] D. P. Bertsekas, *Nonlinear Programming*. Athena Scientific, 1995.

[24] R. Maheswaran and T. Basar, "Decentralized network resource allocation as a repeated noncooperative market game," in *Proceedings of the 40th IEEE Conference on Decision and Control*, vol. 5, 2001, pp. 4565–4570.
[25] G. Scutari, D. Palomar, and S. Barbarossa, "Asynchronous iterative water-filling for gaussian frequency-selective interference channels," *IEEE Transactions on Information Theory*, vol. 54, no. 7, pp. 2868–2878, July 2008.
[26] R. Iltis, S.-J. Kim, and D. Hoang, "Noncooperative iterative MMSE beamforming algorithms for ad hoc networks," *IEEE Transactions on Communications*, vol. 54, no. 4, pp. 748–759, 2006.
[27] W. W. Hager, "Updating the inverse of a matrix," *SIAM Review. A Publication of the Society for Industrial and Applied Mathematics*, vol. 31, no. 2, pp. 221–239, 1989.

APPENDIX I
PROOF OF THEOREM 2

In Theorem 4, we prove that the game converges to a NE. We now need to show that the form of the pricing-factor matrix in (12) and (13) is necessary and sufficient to ensure the achieved NE meets K.K.T optimality conditions of (6).

The achieved NE is characterized by the solutions of all N per-user optimization problems (10). For user i, the Lagrangian function is written as:

$$\begin{aligned}
&L_i(\tilde{\mathbf{T}}_i, \alpha_i^{(k)}, \gamma_i) \\
&= U_i^{'}(\tilde{\mathbf{T}}_i, \tilde{\mathbf{T}}_{-i}) - \sum_{k\in\Psi_K} \alpha_i^{(k)}[\mathrm{tr}(\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}) - P_{mask}(k)] \\
&\quad - \gamma_i[\sum_{k\in\Psi_K} \mathrm{tr}(\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}) - P_{\max}] \\
&= \sum_{k\in\Psi_K} \{R_i^{(k)} - \mathrm{tr}(\tilde{\mathbf{T}}_i^{(k)H}\mathbf{A}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)})\} \\
&\quad - \sum_{k\in\Psi_K} \alpha_i^{(k)}[\mathrm{tr}(\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}) - P_{mask}(k)] \\
&\quad - \gamma_i[\sum_{k\in\Psi_K} \mathrm{tr}(\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}) - P_{\max}]
\end{aligned} \tag{29}$$

where $\alpha_i^{(k)}$ and γ_i are the nonnegative Lagrangian multipliers. Since the individual utility optimization problem is convex, a locally optimal solution is thus globally optimal. The optimal solution can be found by solving its K.K.T. conditions [23], given by:

$$\begin{aligned}
&\frac{\partial L_i(\tilde{\mathbf{T}}_i, \alpha_i^{(k)}, \gamma_i)}{\partial \tilde{\mathbf{T}}_i^{(k)*}} \\
&= \frac{\partial R_i^{(k)}}{\partial \tilde{\mathbf{T}}_i^{(k)*}} - \mathbf{A}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)} - (\alpha_i^{(k)} + \gamma_i)\tilde{\mathbf{T}}_i^{(k)} \\
&= 0,\ \forall\ \mathrm{k} \in \Psi_K \\
&\sum_{k\in\Psi_K} \mathrm{tr}(\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}) - P_{\max} \le 0 \\
&\gamma_i[\sum_{k\in\Psi_K} \mathrm{tr}(\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}) - P_{\max}] = 0 \\
&\mathrm{tr}(\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}) - P_{mask}(k) \le 0,\ \forall\ k \in \Psi_K \\
&\alpha_i^{(k)}[\mathrm{tr}(\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}) - P_{mask}] = 0,\ \forall\ k \in \Psi_K
\end{aligned} \tag{30}$$

The Lagrangian function of the network optimization problem (6) is:

$$\begin{aligned}
&L(\tilde{\mathbf{T}}, \alpha_i^{(k)}, \gamma_i) \\
&= \sum_{i\in\Phi_N} R_{(i)} - \sum_{i\in\Phi_N}\sum_{k\in\Psi_K} \alpha_i^{(k)}[\mathrm{tr}(\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}) - P_{mask}(k)] \\
&\quad - \sum_{i\in\Phi_N} \gamma_i[\sum_{k\in\Psi_K} \mathrm{tr}(\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}) - P_{\max}] \\
&= \sum_{i\in\Phi_N}\sum_{k\in\Psi_K} R_i^{(k)} \\
&\quad - \sum_{i\in\Phi_N}\sum_{k\in\Psi_K} \alpha_i^{(k)}[\mathrm{tr}(\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}) - P_{mask}(k)] \\
&\quad - \sum_{i\in\Phi_N} \gamma_i[\sum_{k\in\Psi_K} \mathrm{tr}(\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}) - P_{\max}]
\end{aligned} \tag{31}$$

where $\tilde{\mathbf{T}} \stackrel{\text{def}}{=} \bigcup_i \tilde{\mathbf{T}}_\mathbf{i}$ is the set of precoding matrices over all users and frequency bands.

All of the stationary or locally optimal points of the network problem must satisfy the its K.K.T. conditions:

$$\begin{aligned}
&\frac{\partial L(\tilde{\mathbf{T}}, \alpha_i^{(k)}, \gamma_i)}{\partial \tilde{\mathbf{T}}_i^{(k)*}} = \frac{\partial R_i^{(k)}}{\partial \tilde{\mathbf{T}}_i^{(k)*}} + \sum_{j\in\Phi_N\setminus\{i\}} \frac{\partial R_j^{(k)}}{\partial \tilde{\mathbf{T}}_i^{(k)*}} - (\alpha_i^{(k)} + \gamma_i)\tilde{\mathbf{T}}_i^{(k)} \\
&\qquad = 0,\ \forall\ k \in \Psi_K,\ \forall\ i \in \Phi_N \\
&\sum_{k\in\Psi_K} \mathrm{tr}(\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}) - P_{\max} \le 0,\ \forall\ i \in \Phi_N \\
&\gamma_i[\sum_{k\in\Psi_K} \mathrm{tr}(\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}) - P_{\max}] = 0,\ \forall\ i \in \Phi_N \\
&\mathrm{tr}(\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}) - P_{mask}(k) \le 0,\ \forall\ k \in \Psi_K,\ \forall\ i \in \Phi_N \\
&\alpha_i^{(k)}[\mathrm{tr}(\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}) - P_{mask}] = 0,\ \forall\ k \in \Psi_K,\ \forall\ i \in \Phi_N
\end{aligned} \tag{32}$$

By comparing (30) and (32), we notice that if the achieved NE meets the K.K.T conditions of the social optimization problem (6) (necessary conditions), the following equality must hold:

$$-\mathbf{A}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)} = \sum_{j\in\Phi_N\setminus\{i\}} \frac{\partial R_j^{(k)}}{\partial \tilde{\mathbf{T}}_i^{(k)*}}. \tag{33}$$

To compute $\frac{\partial R_j^{(k)}}{\partial \tilde{\mathbf{T}}_i^{(k)*}}$, recall (2) and note that:

$$R_j^{(k)} = \log\det(\mathbf{C}_{d(j)}^{(k)} + \mathbf{H}_{d(j),j}^{(k)}\tilde{\mathbf{T}}_j^{(k)}\tilde{\mathbf{T}}_j^{(k)H}\mathbf{H}_{d(j),j}^{(k)H}) - \log\det(\mathbf{C}_{d(j)}^{(k)})$$

and

$$\begin{aligned}
\mathbf{C}_{d(j)}^{(k)} &= \mathbf{I} + \mathbf{H}_{d(j),i}^{(k)}\tilde{\mathbf{T}}_i^{(k)}\tilde{\mathbf{T}}_i^{(k)H}\mathbf{H}_{d(j),i}^{(k)H} \\
&\quad + \sum_{v\in\Phi_N\setminus\{i,j\}} \mathbf{H}_{d(j),v}^{(k)}\tilde{\mathbf{T}}_v^{(k)}\tilde{\mathbf{T}}_v^{(k)H}\mathbf{H}_{d(j),v}^{(k)H}.
\end{aligned}$$

We have

$$\begin{aligned}
\frac{\partial R_j^{(k)}}{\partial \tilde{\mathbf{T}}_i^{(k)*}} &= -\mathbf{H}_{d(j),i}^{(k)H}{\mathbf{C}_{d(j)}^{(k)}}^{-1}\mathbf{H}_{d(j),i}^{(k)}\tilde{\mathbf{T}}_i^{(k)} + \\
&\quad \mathbf{H}_{d(j),i}^{(k)H}(\mathbf{C}_{d(j)}^{(k)} + \mathbf{H}_{d(j),j}^{(k)}\tilde{\mathbf{T}}_j^{(k)}\tilde{\mathbf{T}}_j^{(k)H}\mathbf{H}_{d(j),j}^{(k)H})^{-1}\mathbf{H}_{d(j),i}^{(k)}\tilde{\mathbf{T}}_i^{(k)} \\
&= -\mathbf{H}_{d(j),i}^{(k)H}{\mathbf{C}_{d(j)}^{(k)}}^{-1}\mathbf{H}_{d(j),j}^{(k)}[(\tilde{\mathbf{T}}_j^{(k)}\tilde{\mathbf{T}}_j^{(k)H})^{-1} \\
&\quad + \mathbf{H}_{d(j),j}^{(k)H}{\mathbf{C}_{d(j)}^{(k)}}^{-1}\mathbf{H}_{d(j),j}^{(k)}]^{-1}\mathbf{H}_{d(j),j}^{(k)H}{\mathbf{C}_{d(j)}^{(k)}}^{-1}\mathbf{H}_{d(j),i}^{(k)}\tilde{\mathbf{T}}_i^{(k)}
\end{aligned} \tag{34}$$

The last equality in (34) follows by applying the Woodbury identity [27] to $(\mathbf{C}_{d(j)}^{(k)} + \mathbf{H}_{d(j),j}^{(k)}\tilde{\mathbf{T}}_j^{(k)}\tilde{\mathbf{T}}_j^{(k)H}\mathbf{H}_{d(j),j}^{(k)H})^{-1}$. Plugging (34) into (33), we get (13). It is clear that the derived $\mathbf{A}_i^{(k)}$ matrix is positive-semidefinite and if the pricing-factor has the form (13), the achieved NE meets the K.K.T conditions of problem (6) (sufficient condition). □