A Game Theoretic Design of Artificial-Noise Aided Transmissions in MIMO Wiretap Interference Network

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***Abstract*—The article considers the joint optimization of artificial noise (AN) and information signal precoders in a MIMO wiretap interference network where the transmission of each user may be overheard by several MIMO-capable eavesdroppers. We use the theory of non-cooperative games to propose a distributed framework to optimize the covariance matrices of the information signal and AN at each link. To tackle the non-convexity of each link/player's optimization problem, we recruit a relaxed equilibrium concept in game theory, called *quasi-Nash equilibrium* (QNE). Under the assumption of no coordination between links, we derive sufficient conditions for the existence and uniqueness of the resulting QNE. It turns out that the uniqueness of QNE is not always guaranteed, especially in the case of high interference. Hence, multiple QNEs might exist, and an ordinary updating process (e.g., Gauss-Seidel, Jacobi, or asynchronous update) does not guarantee the convergence to a QNE. Instead, by using the Tikhonov regularization method for variational inequality problems, we modify our algorithm to guarantee the game's convergence to a QNE even in the case of having multiple QNEs. The modified algorithm also allows the links to select between multiple QNEs so as to reduce the received interference at the legitimate receivers. Simulations are then used to confirm the above theoretical findings and the efficacy (in terms of secrecy sum-rate, convergence guarantee, and energy efficiency) of the latter algorithm.**

***Index Terms*—Wiretap interference network, friendly jamming, equilibrium selection, MIMO precoders**

I. INTRODUCTION

Physical-layer (PHY-layer) security provides cost-effective solutions in scenarios where the use of cryptography is either impractical or expensive. Over the last decade, several PHY-layer security techniques have been proposed. Among them is the use of artificial noise (AN) as a friendly jamming (FJ) signal [1]. In this method, along with the information signal, the transmitter and/or its allying radios generate a FJ signal aiming to increase the interference at the eavesdropper (to conceal the information received by the legitimate receiver).

To use AN in a wiretap interference network (e.g. peer-to-peer networks), it is desirable that the FJ interference from each transmitter is cancelled or carefully managed to minimize the effect on other legitimate but unintended receivers in the network. Furthermore, although network interference is regarded as an adverse phenomenon at legitimate receivers, it can be exploited to "blind" eavesdroppers and hence improve the secrecy rate of legitimate transmissions.

In this paper, we consider a MIMO wiretap interference network to assess the potential of interference management in achieving a secrecy target. To that end, we design a non-cooperative game in which the utility of each player (i.e., link) is its secrecy rate and the strategy of each player is to optimize the covariance matrices of information signal and AN. Since the best-response of each player is a non-convex optimization problem, traditional analyses and results of convex (concave) games are not applicable to our game. Instead, we use a relaxed equilibrium concept, known as quasi-Nash equilibrium (QNE) [2]. QNE is a solution of the variational inequality (VI) [3] obtained under the KKT optimality conditions of the players' problems. We first derive sufficient conditions for the existence and uniqueness of the resulting QNE. It turns out that under high network interference, the game might have multiple QNEs. Consequently, the convergence to a QNE is not guaranteed, making our algorithm unstable. To overcome this issue, we use the Tikhonov regularization method used in solving VI problems [4]. Doing so, we define a modified game in which not only the game is guaranteed to converge to a QNE, but also the links are able to choose between the multiple QNEs in the game such that the interference at the legitimate receivers is reduced. Simulations show that the modified algorithm results in more energy efficiency and higher network secrecy rate with guaranteed convergence.

The rest of this paper is organized as follows. In Section II the system model is defined. In Section III the transmit optimization is modeled as a non-cooperative game. The existence and uniqueness of the QNE are investigated in Section IV. The analysis of the game in the presence of multiple equilibria, and theoretical aspects of QNE selection are introduced in Section V. In Section VI the design and implementation aspects of the of QNE selection are discussed. Simulation results are presented in Section VII to verify our theoretical analyses. Finally, Section VIII concludes the paper.

II. System Model

Let us consider a network where Q transmitters (with N_{T_q} antennas at the qth transmitter) are communicating with their corresponding receivers (with N_{R_q} antennas at the qth receiver, $q = 1, \dots, Q$). There are K eavesdroppers each with $N_{e,k}, k = 1, \dots, K$ antennas that can overhear the communications. The received signal at the qth receiver, $\mathbf{y}_q$ is

$$\mathbf{y}_q = \tilde{\mathbf{H}}_{qq}\mathbf{u}_q + \sum_{\substack{r=1 \\ r\neq q}}^{Q} \tilde{\mathbf{H}}_{rq}\mathbf{u}_r + \mathbf{n}_q, \ q = 1, \dots, Q \tag{1}$$

where $\tilde{\mathbf{H}}_{rq}$ denotes the $N_{R_q} \times N_{T_r}$ channel matrix between the rth transmitter and the qth receiver, $\mathbf{u}_q$ is the $N_{T_q} \times 1$ vector of transmitted signal from the qth transmitter, and $\mathbf{n}_q$ is the $N_{R_q} \times 1$ additive noise vector whose elements are i.i.d zero-mean circularly symmetric complex Gaussian distributed with unit variance. The received signal at the kth eavesdropper, $\mathbf{z}_k$ can be expressed as

$$\mathbf{z}_k = \sum_{q=1}^{Q} \mathbf{G}_{qk}\mathbf{u}_q + \mathbf{n}_{e,k}, \ k = 1, \dots, K. \tag{2}$$

where $\mathbf{G}_{qk}$ is the $N_{R_{e,k}} \times N_{T_q}$ channel matrix between the qth transmitter ($q = 1, \dots, Q$) and the kth eavesdropper and $\mathbf{n}_{e,k}$ is the $N_{R_{e,k}} \times 1$ additive noise vector at the receiver of the kth eavesdropper. The transmitted signal $\mathbf{u}_q$ is defined as

$$\mathbf{u}_q \triangleq \mathbf{s}_q + \mathbf{w}_q \tag{3}$$

where $\mathbf{s}_q$ is the information signal and $\mathbf{w}_q$ is the AN. We use the Gaussian codebook for the information signal and a Gaussian noise for the AN. Let $\mathbf{\Sigma}_q$ and $\mathbf{W}_q$ denote the covariance matrices of $\mathbf{s}_q$ and $\mathbf{w}_q$, respectively. The qth link ($q = 1, \dots, Q$) together with K eavesdroppers form a compound wiretap channel. The achievable secrecy rate of the qth link ($q = 1, \dots, Q$) can be written as [5]

$$R_{s,q}(\mathbf{\Sigma}_q, \mathbf{W}_q) \triangleq C_q(\mathbf{\Sigma}_q, \mathbf{W}_q) - \max_{k=1,\dots,K} C_{e,q,k}(\mathbf{\Sigma}_q, \mathbf{W}_q) \tag{4}$$

where $C_q(\mathbf{\Sigma}_q, \mathbf{W}_q)$ is the information rate and $C_{e,q,k}(\mathbf{\Sigma}_q, \mathbf{W}_q)$ is the rate at the kth eavesdropper, $k = 1, \dots, K$, while eavesdropping on the qth link, $q = 1, \dots, Q$. More specifically:

$$C_q(\mathbf{\Sigma}_q, \mathbf{W}_q) \triangleq \ln\left|\mathbf{I} + \mathbf{M}_q^{-1}\mathbf{H}_{qq}\mathbf{\Sigma}_\mathbf{q}\mathbf{H}_{qq}^H\right|, \tag{5a}$$

$$\mathbf{M}_q \triangleq \mathbf{I} + \mathbf{H}_{qq}\mathbf{W}_q\mathbf{H}_{qq}^H + \sum_{\substack{r=1 \\ r\neq q}}^{Q} \mathbf{H}_{rq}\left(\mathbf{\Sigma}_r + \mathbf{W}_r\right)\mathbf{H}_{rq}^H, \tag{5b}$$

$$C_{e,q,k}(\mathbf{\Sigma}_q, \mathbf{W}_q) \triangleq \ln\left|\mathbf{I} + \mathbf{M}_{e,q,k}^{-1}\mathbf{G}_{qk}\mathbf{\Sigma}_\mathbf{q}\mathbf{G}_{qk}^H\right|, \tag{5c}$$

$$\mathbf{M}_{e,q,k} \triangleq \mathbf{I} + \mathbf{G}_{qk}\mathbf{W}_q\mathbf{G}_{qk}^H + \sum_{\substack{r=1 \\ r\neq q}}^{Q} \mathbf{G}_{rk}\left(\mathbf{\Sigma}_r + \mathbf{W}_r\right)\mathbf{G}_{rk}^H, \tag{5d}$$

In the above, the term $\mathbf{M}_q$ is the interference-plus-noise received at the qth receiver, and the term $\mathbf{M}_{e,q,k}$ is the interference-plus-noise received at the kth eavesdropper while eavesdropping on the qth link. Furthermore, we have $Tr(\mathbf{\Sigma}_q + \mathbf{W}_q) \le P_q, \forall q$ indicating the power constraint on each link where $P_q > 0$.

III. Problem Formulation

Due to the lack of coordination amongst links, the dynamics of interactions between them can be modeled as a non-cooperative game where each player (i.e., link) maximizes its utility (i.e., secrecy rate) independently, given the strategies of other players.[1]. The best response problem of each player is formulated as

$$\begin{aligned} &\underset{\mathbf{\Sigma}_q, \mathbf{W}_q}{\text{maximize}} \ R_{s,q}(\mathbf{\Sigma}_q, \mathbf{W}_q) \\ &\text{s.t.} \quad (\mathbf{\Sigma}_q, \mathbf{W}_q) \in \mathcal{F}_q, \ q = 1, \dots, Q \end{aligned} \tag{6}$$

where $\mathcal{F}_q = \{(\mathbf{\Sigma}_q, \mathbf{W}_q) | Tr(\mathbf{\Sigma}_q + \mathbf{W}_q) \le P_q, \ \mathbf{\Sigma}_q \succeq 0, \ \mathbf{W}_q \succeq 0\}$ includes all Hermitian positive (semi)definite matrices that satisfy the power constraint. Problem (6) is in general non-convex. Following the approach of [6], the objective in (6) can be written as

$$\begin{aligned} &\underset{\mathbf{\Sigma}_q, \mathbf{W}_q, \mathbf{S}_q}{\text{maximize}} \ f_q(\mathbf{\Sigma}_q, \mathbf{W}_q, \{\mathbf{S}_{q,k}\}_{k=0}^K), \\ &\text{s.t.} \quad (\mathbf{\Sigma}_q, \mathbf{W}_q) \in \mathcal{F}_q, \ \mathbf{S}_{q,k} \succeq 0, \ \begin{cases} q = 1, \dots, Q \\ k = 0, \dots, K \end{cases} \end{aligned} \tag{7}$$

where

$$f_q(\mathbf{\Sigma}_q, \mathbf{W}_q, \{\mathbf{S}_{q,k}\}_{k=0}^K) \triangleq \varphi_q(\mathbf{\Sigma}_q, \mathbf{W}_q, \mathbf{S}_{q,0}) - \max_{k=1,\dots,K} \varphi_{e,q,k}(\mathbf{\Sigma}_q, \mathbf{W}_q, \mathbf{S}_{q,k}), \tag{8a}$$

$$\varphi_q(\mathbf{\Sigma}_q, \mathbf{W}_q, \mathbf{S}_{q,0}) \triangleq -Tr(\mathbf{S}_{q,0}\mathbf{M}_q) + \ln|\mathbf{S}_{q,0}| + N_{R_q} + \ln\left|\mathbf{M}_q + \mathbf{H}_{qq}\mathbf{\Sigma}_\mathbf{q}\mathbf{H}_{qq}^H\right|, \tag{8b}$$

$$\varphi_{e,q,k}(\mathbf{\Sigma}_q, \mathbf{W}_q, \mathbf{S}_{q,k}) \triangleq Tr(\mathbf{S}_{q,k}(\mathbf{M}_{e,q,k} + \mathbf{G}_{qk}\mathbf{\Sigma}_\mathbf{q}\mathbf{G}_{qk}^H)) - \ln|\mathbf{S}_{q,k}| - N_{R_{e,k}} - \ln|\mathbf{M}_{e,q,k}|, \tag{8c}$$

and $\{\mathbf{S}_{q,k}\}_{k=0}^K = \{\mathbf{S}_{q,0}, \dots, \mathbf{S}_{q,K}\}$ are slack variables defined to convexify the non-convex part of the secrecy rate. It can be easily proved that the function $f_q(\mathbf{\Sigma}_q, \mathbf{W}_q, \{\mathbf{S}_{q,k}\}_{k=0}^K)$ is convex w.r.t. either $(\mathbf{\Sigma}_q, \mathbf{W}_q)$ or $\{\mathbf{S}_{q,k}\}_{k=0}^K$. Therefore, according to [7, Section IV.B], it can be shown that a stationary point to problem (6) that satisfies KKT conditions can be found. That is, in one iteration, problem (7) is solved w.r.t. $\{\mathbf{S}_{q,k}\}_{k=0}^K$ to find an optimal solution $\{\mathbf{S}_{q,k}^*\}_{k=0}^K$. Next, while $\{\mathbf{S}_{q,k}^*\}_{k=0}^K$ is plugged in, problem (7) is solved w.r.t. $(\mathbf{\Sigma}_q, \mathbf{W}_q)$ to find an optimal solution $(\mathbf{\Sigma}_q^*, \mathbf{W}_q^*)$, and this Alternating Optimization (AO) continues until convergence. The nth iteration of AO can be written as

$$(\mathbf{\Sigma}_\mathbf{q}^\mathbf{n}, \mathbf{W}_\mathbf{q}^\mathbf{n}) = \arg\max_{(\mathbf{\Sigma}_q, \mathbf{W}_q)\in\mathcal{F}_q} f_q(\mathbf{\Sigma}_q, \mathbf{W}_q, \left\{\mathbf{S}_{q,k}^{n-1}\right\}_{k=0}^K), \tag{9a}$$

$$\mathbf{S}_{q,0}^n \triangleq \arg\max_{\mathbf{S}_{q,0}\succeq 0} \varphi_q(\mathbf{\Sigma}_q^n, \mathbf{W}_q^n, \mathbf{S}_{q,0}) = (\mathbf{M}_q^n)^{-1} \tag{9b}$$

$$\mathbf{S}_{q,k}^n \triangleq \arg\max_{\mathbf{S}_{q,k}\succeq 0} \varphi_{e,q,k}(\mathbf{\Sigma}_q^n, \mathbf{W}_q^n, \mathbf{S}_{q,k}) = \left(\mathbf{M}_{e,q,k}^n + \mathbf{G}_{qk}\mathbf{\Sigma}_\mathbf{q}{}^n\mathbf{G}_{qk}^H\right)^{-1} \tag{9c}$$

where $n = 1, 2, \dots$. Incorporating (9b) and (9c) in (9a), the solution to the convex problem (9a) can be found using a convex optimization solver (e.g., cvx). According to [8, chapter 3.1.5], the smooth approximation of (6) is

$$\bar{R}_{s,q}(\mathbf{\Sigma}_q, \mathbf{W}_q) = C_q(\mathbf{\Sigma}_q, \mathbf{W}_q) - \frac{1}{\beta}\ln\left(\sum_{k=1}^{K} \exp\{\beta C_{e,q,k}(\mathbf{\Sigma}_q, \mathbf{W}_q)\}\right), \ q = 1, \dots, Q \tag{10}$$

where $\beta > 0$. Hence, we can change (6) to (10) and then do the same reformulation that resulted in (7) to end up with the following smooth reformulation [7]:

[1]Treating the strategies of other players as given corresponds to measuring the interference at the receiver side.

$$\underset{\mathbf{\Sigma}_q,\mathbf{W}_q,\mathbf{S}_q}{\text{maximize}}\ \bar{f}_q(\mathbf{\Sigma}_q,\mathbf{W}_q,\{\mathbf{S}_{q,k}\}_{k=0}^{K}),$$
$$\text{s.t.}\qquad (\mathbf{\Sigma}_q,\mathbf{W}_q)\in\mathcal{F}_q,\ \mathbf{S}_k\succeq 0,\ \begin{cases} q=1,\dots,Q\\ k=0,\dots,K\end{cases} \tag{11}$$

where

$$\bar{f}_q(\mathbf{\Sigma}_q,\mathbf{W}_q,\{\mathbf{S}_{q,k}\}_{k=0}^{K})\triangleq\varphi_q(\mathbf{\Sigma}_q,\mathbf{W}_q,\mathbf{S}_{q,0})- \frac{1}{\beta}\ln(\sum_{k=1}^{K}e^{\beta\varphi_{e,q,k}(\mathbf{\Sigma}_q,\mathbf{W}_q,\mathbf{S}_{q,k})}) \tag{12}$$

with φ_q and $\varphi_{e,q,k}$ defined in (8b) and (8c), respectively. Hence, the AO iteration in (9a) changes to

$$(\mathbf{\Sigma_q^n},\mathbf{W_q^n})=\arg\max_{(\mathbf{\Sigma}_q,\mathbf{W}_q)\in\mathcal{F}_q}\bar{f}_q(\mathbf{\Sigma}_q,\mathbf{W}_q,\{\mathbf{S}_{q,k}^{n-1}\}_{k=0}^{K}), \tag{13}$$

while the solutions for $\left\{\mathbf{S}_{q,k}^{n-1}\right\}_{k=0}^{K}$ (i.e., equations (9b) and (9c)) are already plugged in. The solution to (13) while being at the nth iteration is computed using Projected Gradient (PG) algorithm. The lth iteration of PG algorithm while at the nth iteration of (13) is as follows.

$$\begin{pmatrix}\hat{\mathbf{\Sigma}}_q^{n,l+1}\\ \hat{\mathbf{W}}_q^{n,l+1}\end{pmatrix}=\mathtt{Proj}_{\mathcal{F}_q}\begin{pmatrix}\mathbf{\Sigma}_q^{n,l}+\alpha_l\nabla_{\mathbf{\Sigma}_q}\bar{f}_q^{n,l}\\ \mathbf{W}_q^{n,l}+\alpha_l\nabla_{\mathbf{W}_q}\bar{f}_q^{n,l}\end{pmatrix}, \tag{14}$$

$$\begin{pmatrix}\mathbf{\Sigma}_q^{n,l+1}\\ \mathbf{W}_q^{n,l+1}\end{pmatrix}=\begin{pmatrix}\mathbf{\Sigma}_q^{n,l}\\ \mathbf{W}_q^{n,l}\end{pmatrix}+\varepsilon_l\begin{pmatrix}\hat{\mathbf{\Sigma}}_q^{n,l+1}-\mathbf{\Sigma}_q^{n,l}\\ \hat{\mathbf{W}}_q^{n,l+1}-\mathbf{W}_q^{n,l}\end{pmatrix}, \tag{15}$$

where α_l and ε_l are step sizes that can be determined using Wolfe conditions for PG method; $\mathtt{Proj}_{\mathcal{F}_q}$ is the projection operator to the set $\mathcal{F}_q$ which can be written as

$$\mathtt{Proj}_{\mathcal{F}_q}\begin{pmatrix}\tilde{\mathbf{\Sigma}}\\ \tilde{\mathbf{W}}\end{pmatrix}=\min_{\mathbf{W},\mathbf{\Sigma}\in\mathcal{F}_q}||\mathbf{W}-\tilde{\mathbf{W}}||_F^2+||\mathbf{\Sigma}-\tilde{\mathbf{\Sigma}}||_F^2; \tag{16}$$

and

$$\nabla_{\mathbf{\Sigma}_q}\bar{f}_q^{n,l}=\nabla_{\mathbf{\Sigma}_q}\bar{f}_q(\mathbf{\Sigma}_q^{n,l},\mathbf{W}_q^{n,l},\{\mathbf{S}_{q,k}^{n-1}\}_{k=0}^{K}), \tag{17a}$$
$$\nabla_{\mathbf{W}_q}\bar{f}_q^{n,l}=\nabla_{\mathbf{W}_q}\bar{f}_q(\mathbf{\Sigma}_q^{n,l},\mathbf{W}_q^{n,l},\{\mathbf{S}_{q,k}^{n-1}\}_{k=0}^{K}). \tag{17b}$$

The pseudocode of this game is shown in Algorithm 1.

Algorithm 1 Best-Response Based Secure Transmit Optimization

Initialize: $\mathbf{\Sigma}_q^{1,1}$, $\mathbf{W}_q^{1,1}$, $Tr(\mathbf{\Sigma}_q^{1,1}+\mathbf{W}_q^{1,1})<P_q, \forall q=1,\dots,Q$.

1: **repeat**
2: Each link q computes $\mathbf{M}_q$, $\mathbf{M}_{e,q,k}$ locally, $\forall k=1,\dots,K$.
3:   **for** q = 1,...,Q **do**
4:     **for** n = 1,... **do**
5:       Compute $\mathbf{S}_{q,k}^{n-1}$, $k=0,\dots,K$.
6:       **for** l = 1,... **do**
7:         Compute $\varphi_{e,q,k}$, $\mathbf{M}_q^{n,l}$, and $\mathbf{M}_{e,q,k}^{n,l}$, $\forall(q,k)$.
8:         Compute $(\mathbf{\Sigma}_q^{n,l+1},\mathbf{W}_q^{n,l+1})$ using (14) and (15).
9:       **end for**
10:     **end for**
11:   **end for**
12: **until** Convergence to QNE.

IV. Existence and Uniqueness of the QNE

Before studying the existence and uniqueness of NE, we review the fundamentals of variational inequality theory [3]:

Variational Inequality (VI) Theory: Let $F:\mathcal{Q}\to\mathbb{R}^n$ be a vector-valued continuous real function, where $\mathcal{Q}\subseteq\mathbb{R}^n$ is a nonempty, closed, and convex set. The variational inequality VI$(F,\mathcal{Q})$ is the problem of finding a vector x^* such that

$$(x-x^*)^T F(x^*)\geq 0,\quad \forall x\in\mathcal{Q}. \tag{18}$$

The NE for a non-cooperative game can be introduced as the solution of a VI [4, Chapter 2].

The optimization problem of each player mentioned in (10) is non-convex. Hence, the solution found for each link at line 10 of Algorithm 1 is only a stationary point of problem (10). As a consequence, the traditional concepts introduced in concave (convex) games to prove the existence of a NE are not applicable here. Instead, we analyze the proposed (non-convex) smooth game based on the relaxed equilibrium concept of QNE [2]. The QNE is by definition a tuple that satisfies the KKT conditions of all players' optimization problems. Under a constraint qualification, stationary points of each player's optimization problem satisfies its KKT conditions. To begin the analysis of the QNE, we first show an important property of the stationary point found using the AO technique (i.e., line 4-10 of Algorithm 1).

Proposition 1. *The stationary point of problem* (11) *found using Algorithm 1 satisfies the KKT conditions of*

$$\underset{\mathbf{\Sigma}_q,\mathbf{W}_q}{\textit{maximize}}\ \bar{R}_{s,q}(\mathbf{\Sigma}_q,\mathbf{W}_q)$$
$$\textit{s.t.}\qquad (\mathbf{\Sigma}_q,\mathbf{W}_q)\in\mathcal{F}_q,\ q=1,\dots,Q. \tag{19}$$

Proof: See [6, Appendix A]. ∎

Converting the game in (11) to a VI is skipped for brevity (see [6, Section IV]). Consider the following function:

$$F^{\mathbb{C}}=\left[F_1^{\mathbb{C}}(\mathbf{\Sigma}_1,\mathbf{W}_1,\{S_{1,k}\}_{k=0}^{K})^T,\dots,F_Q^{\mathbb{C}}(\mathbf{\Sigma}_Q,\mathbf{W}_Q,\{S_{Q,k}\}_{k=0}^{K})^T\right]\triangleq \left[\left[-(\nabla_{\mathbf{\Sigma}_1}\bar{f}_1)^T,-(\nabla_{\mathbf{W}_1}\bar{f}_1)^T\right]^T,\dots,\left[-(\nabla_{\mathbf{\Sigma}_Q}\bar{f}_Q)^T,-(\nabla_{\mathbf{W}_Q}\bar{f}_Q)^T\right]^T\right]. \tag{20}$$

Furthermore, let $vec(Z)=[(Z_{:,1})^T,\dots,(Z_{:,N})^T]^T$ be the vectorized version of the complex matrix Z, where $Z_{:,n}$ indicates the nth column of matrix Z. Assuming that $(\bar{\mathbf{\Sigma}},\bar{\mathbf{W}})\triangleq(\bar{\mathbf{\Sigma}}_1,\bar{\mathbf{W}}_1,\dots,\bar{\mathbf{\Sigma}}_Q,\bar{\mathbf{W}}_Q)$ is the QNE to the transmit optimization game, the equivalent VI is as follows

$$\left(\left[\mathbf{\Sigma}^{\mathbb{R}T},\mathbf{W}^{\mathbb{R}T}\right]-\left[\bar{\mathbf{\Sigma}}^{\mathbb{R}T},\bar{\mathbf{W}}^{\mathbb{R}T}\right]\right)F^{\mathbb{R}}\geq 0,\ \forall(\mathbf{\Sigma}^{\mathbb{R}},\mathbf{W}^{\mathbb{R}})\in\mathcal{K}^{\mathbb{R}}, \tag{21}$$

where

$$\bar{\mathbf{\Sigma}}^{\mathbb{R}}\triangleq[Re\{vec(\bar{\mathbf{\Sigma}})\}^T,Im\{vec(\bar{\mathbf{\Sigma}})\}^T]^T, \tag{22}$$
$$\bar{\mathbf{W}}^{\mathbb{R}}\triangleq[Re\{vec(\bar{\mathbf{W}})\}^T,Im\{vec(\bar{\mathbf{W}}\}^T]^T, \tag{23}$$
$$F^{\mathbb{R}}\triangleq[Re\{vec(F^{\mathbb{C}})\}^T,Im\{vec(F^{\mathbb{C}}\}^T]^T \tag{24}$$

and $Re\{\}$ and $Im\{\}$ indicating the real and imaginary parts, respectively. The terms $F^{\mathbb{R}}$, $\bar{\mathbf{\Sigma}}^{\mathbb{R}}$, and $\bar{\mathbf{W}}^{\mathbb{R}}$ are the vectorized versions of $F^{\mathbb{C}}$, $\bar{\mathbf{\Sigma}}$, and $\bar{\mathbf{W}}$, respectively. The set $\mathcal{K}^{\mathbb{R}}$ includes the vectorized versions of $(\bar{\mathbf{\Sigma}},\bar{\mathbf{W}})\triangleq(\bar{\mathbf{\Sigma}}_1,\bar{\mathbf{W}}_1,\dots,\bar{\mathbf{\Sigma}}_Q,\bar{\mathbf{W}}_Q)$. In the following theorem, the existence of QNE is proved.

Theorem 1. *The proposed game, where the actions of each player is given by* (11) *admits at least one QNE.*

Proof: See [6, Appendix B]. ∎

The uniqueness of QNE is discussed in the following:

Theorem 2. *A sufficient condition for the game described by the VI in* (21) *to have a unique QNE is*

$$\lambda_{q,\min}>\sum_{\substack{q=1\\ q\neq l}}^{Q}|||D_{Z_l}F_q^{\mathbb{C}}(Z_q)|||_2,\ q=1,\dots,Q \tag{25}$$

where $\lambda_{q,\min}$ is the smallest eigenvalue of $D_{Z_q} F_q^{\mathbb{C}}(Z_q)$, $Z_q = (\boldsymbol{\Sigma}_q, \mathbf{W}_q)$, and $D_{Z_l} F_q^{\mathbb{C}}(Z_q) \triangleq \frac{\partial\, vec\left(F_q^{\mathbb{C}}(Z_q)\right)}{\partial\, vec(Z_l)^T}$ for all $q, l \in \{1, ..., Q\}^2$ or alternatively

$$D_{Z_l} F_q^{\mathbb{C}}(Z_q) \triangleq \begin{bmatrix} D_{\boldsymbol{\Sigma}_l}(-\nabla_{\boldsymbol{\Sigma}_q} \bar{f}_q) & D_{\mathbf{W}_l}(-\nabla_{\boldsymbol{\Sigma}_q} \bar{f}_q) \\ D_{\boldsymbol{\Sigma}_l}(-\nabla_{\mathbf{W}_q} \bar{f}_q) & D_{\mathbf{W}_l}(-\nabla_{\mathbf{W}_q} \bar{f}_q) \end{bmatrix}. \quad (26)$$

Proof: See [6, Appendix C]. ■

V. Transmit Optimization Game with Multiple Equilibria

For the cases where the QNE is not unique (e.g., high interference), Theorem 2 may not hold, and the performance of the network varies significantly over the set of quasi-Nash equilibria. Because of this reason, we propose an alternative algorithm so as to not only guarantee the convergence to a QNE, but also pave the way for further performance improvements.

A. Gradient Response Algorithm

A solution to the VI in (21) can be characterized by the following iteration [4, Chapter 12]:

$$x^{(i+1)} = \Pi_{\mathcal{K}^{\mathbb{R}}} \left(x^{(i)} - \gamma F^{\mathbb{R}}(x^{(i)}, \{\mathbf{S}_{q,k}^{(i)}\}_{k=0}^{K}) \right) \quad (27)$$

where $\Pi_{\mathcal{K}^{\mathbb{R}}}$ is the projection to set $\mathcal{K}^{\mathbb{R}}$, $x = \left[\boldsymbol{\Sigma}^{\mathbb{R}^T}, \mathbf{W}^{\mathbb{R}^T} \right]^T$, the superscript (i) is the number of iterations, and $\gamma = \text{diag}([\gamma_1, \ldots, \gamma_N]^T)$ is a diagonal matrix which indicates the step-size each player takes in the improving direction of its utility function. The solutions to $\{\mathbf{S}_{q,k}^{(i)}\}_{k=0}^{K}$ are as follows:

$$\mathbf{S}_{q,0}^{(i)} \triangleq (\mathbf{M}_q^{(i)})^{-1} = (\mathbf{I} + \mathbf{H}_{qq} \mathbf{W}_q^{(i)} \mathbf{H}_{qq}^H + \sum_{\substack{r=1 \\ r \neq q}}^{Q} \mathbf{H}_{rq} (\boldsymbol{\Sigma}_r^{(i-1)} + \mathbf{W}_r^{(i-1)}) \mathbf{H}_{rq}^H)^{-1}, \quad (28a)$$

$$\mathbf{S}_{q,k}^{(i)} \triangleq (\mathbf{M}_{e,q,k}^{(i)} + \mathbf{G}_{qk} \boldsymbol{\Sigma}_{\mathbf{q}}{}^{(i)} \mathbf{G}_{qk}^H)^{-1} = (\mathbf{I} + \mathbf{G}_{qk} (\boldsymbol{\Sigma}_q^{(i)} + W_q^{(i)}) \mathbf{G}_{qk}^H + \sum_{\substack{r=1 \\ r \neq q}}^{Q} \mathbf{G}_{rk} (\boldsymbol{\Sigma}_r^{(i-1)} + \mathbf{W}_r^{(i-1)}) \mathbf{G}_{rk}^H)^{-1}. \quad (28b)$$

The only difference of the iteration in (27) from Algorithm 1 is that at each round of the game, instead of iteratively performing the AO iterations until the optimum point, each user only does one iteration of the PG method and one iteration of AO, meaning that once one iteration of PG method is done, the values of $\{\mathbf{S}_{q,k}^{(i)}\}_{k=0}^{K}$ will be updated according to (28a) and (28b), and the links start transmission. Notice that since the values of $\{\mathbf{S}_{q,k}^{(i)}\}_{k=0}^{K}$ are uniquely determined for a given $x^{(i)}$, we drop the term $\{\mathbf{S}_{q,k}^{(i)}\}_{k=0}^{K}$ from the argument of $F^{\mathbb{R}}$.

Assuming that $F^{\mathbb{R}}$ is *strongly monotone*[2] (with a modulus $c_{sm}/2$) and Lipschitz continuous (with constant L) w.r.t. $(\boldsymbol{\Sigma}_q, \mathbf{W}_q)$[3], the convergence follows (i.e., QNE is reached) if $\gamma_{i'} = d < \frac{c_{sm}}{L^2}$, $\forall i' = 1, \ldots, N$. Hence, the mapping in (27) becomes a contraction mapping and the fixed points of this map are solutions of the VI in (21) [4, Chapter 12].

The iteration proposed in (27) has two major issues. First, the Lipschitz constant of $F^{\mathbb{R}}(x)$ has to be known. Apart from being difficult to derive, the knowledge of Lipschitz constant requires the global knowledge of best responses in the network. Second, the strong monotonicity of $F^{\mathbb{R}}$ cannot be always guaranteed, meaning that at some point, the game might have more than one equilibria, and the iteration in (27) might not converge. These issues are addressed in the next subsections.

B. Tikhonov Regularization

In Tikhonov regularization, the process of regularizing VI($F^{\mathbb{R}}, \mathcal{K}^{\mathbb{R}}$) involves solving a sequence of VIs, where the following is characterized for a given ϵ [4, chapter 12]:

$$x^{(i+1)} = \Pi_{\mathcal{K}^{\mathbb{R}}} \left(x^{(i)} - \gamma \left(F^{\mathbb{R}}(x^{(i)}) + \epsilon x^{(i)} \right) \right). \quad (29)$$

The solution to this iteration when $i \to \infty$ is denoted as $x(\epsilon)$. The limit point of a sequence of solutions for the modified VI will converge to some solution of the original VI as $\epsilon \to 0$, hence guiding the game to a single QNE among multiple QNEs. That is, we can construct a nested algorithm in which for the jth iteration of outer loop we choose $\epsilon^{(j)}$. Then, we use (29) as the inner loop iteration. The solution of inner loop for a given $\epsilon^{(j)}$ is denoted by $x(\epsilon^{(j)})$ which is the jth member of sequence of solutions to $VI(F^{\mathbb{R}}(x^{(i)}) + \epsilon^{(j)} x, \mathcal{K}^{\mathbb{R}})$. The sequence of ϵ^j's satisfy $\lim_{j \to \infty} \epsilon^{(j)} = 0$. Assuming that $F^{\mathbb{R}}$ is monotone, solving a sequence of $VI(F^{\mathbb{R}}(x) + \epsilon x, \mathcal{K}^{\mathbb{R}})$'s (via (29)) has a limit point (i.e., $\lim_{\epsilon \to 0} x(\epsilon)$ exists) which is the least-norm solution of the VI($F^{\mathbb{R}}, \mathcal{K}^{\mathbb{R}}$) [4, Theorem 12.2.3].

C. Equilibrium Selection using Tikhonov Regularization

We wish to select a QNE that optimizes a particular criterion. Let the set of solutions of VI($F^{\mathbb{R}}, \mathcal{K}^{\mathbb{R}}$) be denoted as SOL($F^{\mathbb{R}}, \mathcal{K}^{\mathbb{R}}$). The selected QNE minimizes a strongly convex function[4] $\Phi(x) : \mathcal{K}^{\mathbb{R}} \to \mathbb{R}$, i.e., the selected QNE solves

$$\begin{aligned} &\text{minimize} \quad \Phi(x) \\ &\text{s.t.} \quad x \in \text{SOL}(F^{\mathbb{R}}, \mathcal{K}^{\mathbb{R}}). \end{aligned} \quad (30)$$

The unique point that solves the optimization in (30), is the solution to VI($\nabla\Phi(x)$, SOL($F^{\mathbb{R}}, \mathcal{K}^{\mathbb{R}}$)). As there is no prior knowledge of the set SOL($F^{\mathbb{R}}, \mathcal{K}^{\mathbb{R}}$) (i.e., no access to all of QNEs), the optimization in (30) cannot be solved easily. Hence, we make the following modification to VI($F^{\mathbb{R}}, \mathcal{K}^{\mathbb{R}}$):

$$F_{\epsilon}^{\mathbb{R}} \triangleq F^{\mathbb{R}} + \epsilon \nabla\Phi(x). \quad (31)$$

Since $\Phi(x)$ is a strongly convex function, its derivative w.r.t. x is strongly monotone. If $F^{\mathbb{R}}$ is monotone[5], then $F_{\epsilon}^{\mathbb{R}}$ is strongly monotone and the solution to VI($F_{\epsilon}^{\mathbb{R}}, \mathcal{K}^{\mathbb{R}}$), namely, $x(\epsilon)$, is unique $\forall \epsilon > 0$. Now, we can use the following iteration

$$x^{(i+1)} = \Pi_{\mathcal{K}^{\mathbb{R}}} \left(x^{(i)} - \gamma \left(F^{\mathbb{R}}(x^{(i)}) + \epsilon \nabla\Phi(x^{(i)}) \right) \right) \quad (32)$$

[2] The notion of (strong) monotonicity is a basic definition in the topic of VI, which is skipped due to space limitations (see [9] and [4]).

[3] It can be seen from (14) and (15) that the power constraint of each user makes the gradients bounded. Hence, $F^{\mathbb{R}}$ is Lipschitz continuous on $\mathcal{K}^{\mathbb{R}}$.

[4] A strongly convex function is a function whose derivative is strongly monotone (see [3]).

[5] Throughout the simulations, it turns out that $F^{\mathbb{R}}$ is always monotone (see [6, Section VII.C]).

where the only difference with (29) is that the multiplier of ϵ in (29) is replaced by $\nabla\Phi(x^{(i)})$. The following theorem shows the potential of using (32) in QNE selection.

Theorem 3. *[4, pp. 1128 and Theorem 12.2.5] Consider the VI problem VI($F_\epsilon^{\mathbb{R}}, \mathcal{K}^{\mathbb{R}}$) with $x(\epsilon)$ as its solution. Assume that the set $\mathcal{K}^{\mathbb{R}}$ is closed and convex, and SOL($F^{\mathbb{R}}, \mathcal{K}^{\mathbb{R}}$) is nonempty. Hence, the limit point of the solutions found via* (32) *(i.e., $\lim_{\epsilon\to 0} x(\epsilon)$ where $x(\epsilon)$ is found by iteratively using* (32) *for $i \to \infty$) is the unique solution of VI($\nabla\Phi(x)$, SOL($F^{\mathbb{R}}, \mathcal{K}^{\mathbb{R}}$)).* □

D. Distributed Tikhonov Regularization

QNE selection requires to be done in two nested loops. In the inner loop, for a given $\epsilon^{(j)}$, the solution to VI($F_\epsilon^{\mathbb{R}}, \mathcal{K}^{\mathbb{R}}$) will be found from the iteration in (32). In the outer loop, the next value of $\epsilon^{(j)}$ will be chosen (where $\lim_{j\to\infty} \epsilon^{(j)} = 0$) until the solution to $\mathrm{VI}(\nabla\Phi(x), \mathrm{SOL}(F^{\mathbb{R}}, \mathcal{K}^{\mathbb{R}}))$ is reached. We introduce another method of regularization, namely, *proximal point regularization*, as the next level of perturbation done on the users' utility functions. The basic perturbation that is done here is to add a term $\theta^{(i)}(x^{(i)} - x^{(i-1)})$ to the function $F_\epsilon^{\mathbb{R}}(x)$ to build a function $F_{\epsilon,\theta}^{\mathbb{R}}(x) \triangleq F_\epsilon^{\mathbb{R}}(x) + \theta^{(i)}(x^{(i)} - x^{(i-1)})$ where $\theta^{(i)}$ is a diagonal matrix. Notice that the superscript (i) indicates the iterations of gradient response algorithm in (27) (or (32)). Hence, the iterates of $x^{(i)}$ can be written as

$$x^{(i+1)} = \Pi_{\mathcal{K}^{\mathbb{R}}}\left(x^{(i)} - \gamma^{(i)}\left(F^{\mathbb{R}}(x^{(i)}) + \epsilon^{(j)}\nabla\Phi(x) + \theta^{(i)}(x^{(i)} - x^{(i-1)})\right)\right) \tag{33}$$

Furthermore, the following property can be proved:

Proposition 2. *[10, Proposition 3.4] Consider the mapping $F_\epsilon^{\mathbb{R}}(x)$ to be a strictly monotone and Lipschitz continuous mapping[6]; $\max_{z\in\mathcal{K}^{\mathbb{R}}} ||x|| \leq C$ and $\max_{z\in\mathcal{K}^{\mathbb{R}}} ||F_\epsilon^{\mathbb{R}}|| \leq B$. Furthermore, suppose that for a given $\epsilon^{(j)}$, the solution to VI($F_\epsilon^{\mathbb{R}}, \mathcal{K}^{\mathbb{R}}$) is denoted as $x(\epsilon^{(j)})$. Let $x^{(i)}$ denote the set of iterates defined by equation* (33) *where the step-size matrix $\gamma^{(i)}$ is changing with the iterations. Lastly, set $\gamma^{(i)}\theta^{(i)} = c = diag([c_1, \ldots, c_N])$ where $c_{i'} \in (0,1), \forall i' = 1, \ldots, N$ is a constant, and let the following hold:*

$$\sum_{i=1} \gamma_{max}^{(i)} = \infty, \sum_{i=1}^{\infty}\left(\gamma_{max}^{(i)}\right)^2 < \infty, \sum_{i=1}^{\infty}(\gamma_{max}^{(i)} - \gamma_{min}^{(i)}) < \infty \tag{34}$$

where $\gamma_{max}^{(i)}$ and $\gamma_{min}^{(i)}$ are respectively the maximum and minimum diagonal elements of the matrix $\gamma^{(i)}$. Therefore, we have $\lim_{i\to\infty} x^{(i)} = x(\epsilon^{(j)})$. □

Note that the strict monotonicity of $F_\epsilon^{\mathbb{R}}(x)$ is satisfied as $F_\epsilon^{\mathbb{R}}(x)$ is already strongly monotone (cf. (31)). By having power constraints on the users' beamformers, the conditions $\max_{z\in\mathcal{K}^{\mathbb{R}}} ||x|| \leq C$ and $\max_{z\in\mathcal{K}^{\mathbb{R}}} ||F_\epsilon^{\mathbb{R}}|| \leq B$ can also be satisfied. According to [10, Proposition 3.4], the step-size $\gamma^{(i)}$ can be chosen as $\gamma_m^{(i)} = (i + \alpha_m)^{-\omega}$, where α_m is a positive integer for $m = 1, \ldots, N$, and $0 < \omega < 1$. With Proposition 2, the Lipschitz constant and strong monotonicity modulus of $F_\epsilon^{\mathbb{R}}$ are not needed to be known.

VI. Algorithm Design

The pseudocode for the QNE selection algorithm is shown in Algorithm 2.

[6]Note that Lipschitz continuity of $F_\epsilon^{\mathbb{R}}(x)$ requires both $F^{\mathbb{R}}(x)$ and $\nabla\Phi(x)$ to be Lipschitz continuous. Hence, the criterion function that we choose in the next section is also Lipschitz continuous.

Algorithm 2 QNE Selection Algorithm

Initialize: $\mathbf{\Sigma}_q^{(1)}$, $\mathbf{W}_q^{(1)}$, $Tr(\mathbf{\Sigma}_q^{(1)} + \mathbf{W}_q^{(1)}) < P_q$, $\forall q$, and j = 1.
1: **repeat** % Outer loop: superscript (j) indicates the iterations starting from here.
2: Choose the jth member of the sequence $\epsilon^{(j)}$.
3: **repeat** % Inner Loop: superscript (i) indicates the iterations starting from here.
4: Compute $\mathbf{M}_q^{(i)}$, $\mathbf{M}_{e,q,k}^{(i)}$, $\forall(q,k) \in \{1,\ldots,Q\} \times \{1,\ldots,K\}$.
5: Compute $\mathbf{S}_{q,k}^{(i)}$, $\forall(q,k) \in \{1,\ldots,Q\} \times \{1,\ldots,K\}$.
6: Compute $\varphi_{e,q,k}(\mathbf{\Sigma}_q^{(i)}, \mathbf{W}_q^{(i)}, \mathbf{S}_{q,k}^{(i)})$, $\forall(q,k) \in \{1,\ldots,Q\} \times \{1,\ldots,K\}$.
7: **for** q = 1,…,Q **do**
8: Replace $\nabla_{\mathbf{\Sigma}_q}\bar{f}_q$ with $\nabla_{\mathbf{\Sigma}_q}\bar{f}_q + \epsilon^{(j)}\nabla_{\mathbf{\Sigma}_q}\Phi(x) - \theta_q^{(i)}\left(\mathbf{\Sigma}_q^{(i)} - \mathbf{\Sigma}_q^{(i-1)}\right)$.
9: Replace $\nabla_{\mathbf{W}_q}\bar{f}_q$ with $\nabla_{\mathbf{W}_q}\bar{f}_q + \epsilon^{(j)}\nabla_{\mathbf{W}_q}\Phi(x) - \theta_q^{(i)}\left(\mathbf{W}_q^{(i)} - \mathbf{W}_q^{(i-1)}\right)$.
10: Compute $(\mathbf{\Sigma}_q^{(i+1)}, \mathbf{W}_q^{(i+1)})$ using (14)-(15). % Drop the superscript n and replace the superscript l with (i). The computation is done the same as (27). Also, in (14) and (15), set $\alpha_l = \gamma_q^{(i)}$ and $\varepsilon_l = 1$.
11: **end for**
12: **until** Convergence to NE. % $x(\epsilon^j)$ is found.
13: j = j+1.
14: **until** Convergence to limit point of $x(\epsilon^j)$.

In the following, we explain the terms $\nabla_{\mathbf{\Sigma}_q}\Phi(x)$ and $\nabla_{\mathbf{W}_q}\Phi(x)$ in lines 8 and 9 of Algorithm 2.

Assume that the criterion function is described as

$$\nabla\Phi(x) \triangleq [\nabla_{\mathbf{\Sigma}_1,\mathbf{W}_1}^{\mathbb{R}}\Phi(x)^T, \ldots, \nabla_{\mathbf{\Sigma}_Q,\mathbf{W}_Q}\Phi(x)^T]^T, \tag{35a}$$

$$\nabla_{\mathbf{\Sigma}_q,\mathbf{W}_q}^{\mathbb{R}}\Phi(x) \triangleq [\nabla_{\mathbf{\Sigma}_q}^{\mathbb{R}}\Phi(x)^T, \nabla_{\mathbf{W}_q}^{\mathbb{R}}\Phi(x)^T]^T,\ q = 1, \ldots, Q, \tag{35b}$$

$$\nabla_{\mathbf{\Sigma}_q}^{\mathbb{R}}\Phi(x) \triangleq [Re(vec(\nabla_{\mathbf{\Sigma}_q}\Phi(x)))^T, Im(vec(\nabla_{\mathbf{\Sigma}_q}\Phi(x)))^T]^T, \tag{35c}$$

$$\nabla_{\mathbf{W}_q}^{\mathbb{R}}\Phi(x) \triangleq [Re(vec(\nabla_{\mathbf{W}_q}\Phi(x)))^T, Im(vec(\nabla_{\mathbf{W}_q}\Phi(x)))^T]^T. \tag{35d}$$

We aim to select the QNE that maximizes the sum-rate of the links. Recalling the reformulated information rate in (8b), $\Phi(x)$ can be described by (with $q \in \mathbb{Q}$):

$$\nabla_{\mathbf{\Sigma}_q}\Phi(x) = \sum_{\substack{r=1\\r\neq q}}^{Q}\mathbf{H}_{qr}^H\left((\mathbf{M}_r + \mathbf{H}_{rr}\mathbf{\Sigma}_r\mathbf{H}_{rr}^H)^{-1} - \mathbf{S}_{r,0}\right)\mathbf{H}_{qr}, \tag{36a}$$

$$\nabla_{\mathbf{W}_q}\Phi(x) = \sum_{\substack{r=1\\r\neq q}}^{Q}\mathbf{H}_{qr}^H\left((\mathbf{M}_r + \mathbf{H}_{rr}\mathbf{\Sigma}_r\mathbf{H}_{rr}^H)^{-1} - \mathbf{S}_{r,0}\right)\mathbf{H}_{qr}. \tag{36b}$$

Notice that although we wrote Φ as a function of x, one can easily relate the vector x to the covariance matrices $\{(\mathbf{\Sigma}_q, \mathbf{W}_q)\}_{q=1}^Q$. We saw in Theorem 3 that the limit point $\lim_{j\to\infty} x(\epsilon^{(j)})$ exists. Hence, the derivatives of $\Phi(x)$ in the limit point of $x(\epsilon^{(j)})$ would be (with $q \in \mathbb{Q}$)

$$\nabla_{\mathbf{\Sigma}_q}\Phi(x) = \sum_{\substack{r=1\\r\neq q}}^{Q}\mathbf{H}_{qr}^H\left((\mathbf{M}_r^\star + \mathbf{H}_{rr}\mathbf{\Sigma}_r^\star\mathbf{H}_{rr})^{-1} - \mathbf{S}_{r,0}^\star\right)\mathbf{H}_{qr} \tag{37a}$$

$$\nabla_{\mathbf{W}_q}\Phi(x) = \sum_{\substack{r=1\\r\neq q}}^{Q}\mathbf{H}_{qr}^H\left((\mathbf{M}_r^\star + \mathbf{H}_{rr}\mathbf{\Sigma}_r^\star\mathbf{H}_{rr})^{-1} - \mathbf{S}_{r,0}^\star\right)\mathbf{H}_{qr} \tag{37b}$$

where $\mathbf{M}_r^\star = \mathbf{I} + \mathbf{H}_{rr}(\mathbf{W}_r^\star)\mathbf{H}_{rr}^H + \mathbf{H}_{qr}(\mathbf{W}_q^\star + \mathbf{\Sigma}_q^\star)\mathbf{H}_{qr}^H + \sum_{\substack{l=1\\l\neq q,r}}^{Q}\mathbf{H}_{lr}\left(\mathbf{\Sigma}_l^\star + \mathbf{W}_l^\star\right)\mathbf{H}_{lr}^H$, with $\mathbf{\Sigma}_q^\star$ and $\mathbf{W}_q^\star$ being the limit points of Algorithm 2. Integrating (37a) w.r.t. $\mathbf{\Sigma}_q^\star$ and integrating (37b) w.r.t. $\mathbf{W}_q^\star$, we end up with $\Phi(x) = \sum_{q=1}^{Q}\sum_{\substack{r=1\\r\neq q}}^{Q}\varphi_r(\mathbf{\Sigma}_r, \mathbf{W}_r, \mathbf{S}_{r,0})$ where $\varphi_r(\mathbf{\Sigma}_r, \mathbf{W}_r, \mathbf{S}_{r,0})$ is defined in (8b). Hence, in the limit point of Algorithm 2, the QNE that maximizes sum-rate of other users is selected.

VII. Numerical Results and Discussion

In this section, we simulate both algorithms presented so far. We set the noise power to 0 dBm. The links are randomly placed in a circle, namely, the simulation region, with a radius r_{circ}. The eavesdroppers are randomly placed within this circle. The distance between the transmitter and the receiver of each link is constant, $d_{link} = 10$ m. The path-loss exponent is set to 2.5. For both algorithms, the termination criterion, wherever it exists, is when the normalized relative difference of each link's secrecy rate between two consecutive iterations is less than 10^{-3}. For the case of QNE selection, we set the parameters according to the following: $\gamma_q^{(i)} = \text{diag}(\gamma_0 i^{(-0.01)})$, $\gamma_0 = 20000$, $c = 0.08 \times I_{Q\times Q}$, and $\epsilon^{(j)} = \frac{1}{j}$.

Fig. 1 shows the convergence behavior of Algorithm 1 and Algorithm 2 for the case of high interference. Due to the existence of multiple QNEs, Algorithm 1 never converges to a point. We increased the number of iterations to 1000, but did not see the convergence of Algorithm 1. However, Algorithm 2 converges to a unique QNE.

Fig. 2 compares the secrecy sum-rate of Algorithm 1 and Algorithm 2 for different number of links. It can be seen that Algorithm 2 consistently outperforms Algorithm 1 in terms of secrecy sum-rate. The reduction in the performance of Algorithm 1 is due to high interference which directly affects the information rate of the legitimate channel.

In Fig. 3, we compare the power efficiency of the proposed algorithms. The total power and total AN powers consumed by the links are normalized w.r.t the total power budget of all links. It can be seen that Algorithm 2 is more energy efficient. Algorithm 1 is less efficient as the increase in the power of AN creates interference at other legitimate receivers which eventually does not lead to a higher secrecy sum-rate[7].

VIII. Conclusions

We designed a game theoretic secure transmit optimization framework for a MIMO interference network with several MIMO-enabled eavesdroppers. To guarantee the convergence and improve the network secrecy rate in the case of multiple QNEs, we designed an algorithm based on the concept of variational inequality with which the links can select the best QNE according to a criterion function. We also verified the efficacy of the QNE selection algorithm by simulations.

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[7]We refer the interested reader to [6, Section VII.C and Section VIII] for a thorough comparison of signaling overhead and other simulation scenarios.

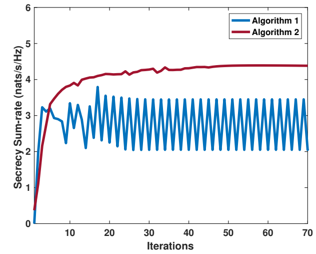


Fig. 1: Convergence of the proposed algorithms for multiple QNE case: $Q = 8, K = 5, N_{T_q} = 5, N_{r_q} = 2, N_{e,k} = 2, r_{circ} = 20$ m, $P_q = 20$ dBm.

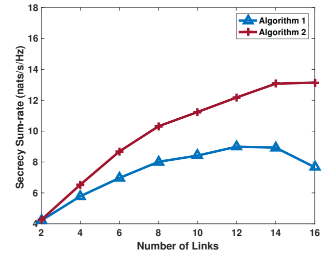
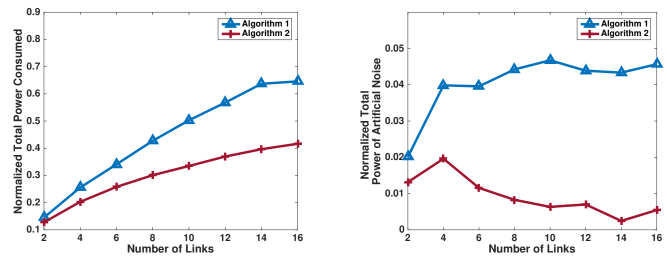


Fig. 2: Comparison of secrecy sum-rate vs. number of links: $N_{T_q} = 5, N_{r_q} = 2, N_{e,k} = 2, r_{circ} = 30$ m, $K = 5, P_q = 40$ dBm.



a) b)

Fig. 3: Comparison of a) total power b) total power of artificial noise: $r_{circ} = 30$ m, $K = 5, N_{T_q} = 5, N_{r_q} = 2, N_{e,k} = 2, P_q = 40$ dBm.

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